

ERRATA for T. Shifrin's *Multivariable Mathematics: Linear Algebra, Multivariable Calculus, and Manifolds*

- p. 47, line 11.** In the rightmost determinant, the first entry of the second column should be z_1 .
- p. 79, Exercise 4.** x should be \mathbf{x} throughout.
- p. 86, Exercise 12b.** Here $\mathbf{f}: \mathcal{M}_{n \times n} \rightarrow \mathbb{R}$.
- p. 91, Example 5.** On the second line it should be $\mathbf{f}: \mathbb{R}^2 - \{y = 0\} \rightarrow \mathbb{R}^2$.
- p. 92, Example 7.** The reference should be to Example 3 of Chapter 2, Section 3.
- p. 93, Proposition 2.4, ff.** Standard terminology is that a function f is \mathcal{C}^1 if f and its partial derivatives are continuous. Note that in the proof of the Proposition, since the partial derivatives exist, we get continuity of f along horizontal and vertical lines, which is all we need to apply the Mean Value Theorem. Thus, the Proposition is correct as stated.
- p. 103, Exercise 6.** The symbol for liter (l) looks too much like a 1. For clarity, it would help to change these to ℓ .
- p. 103, Exercise 11.** Prove that a *differentiable* function f is homogeneous ...
- p. 145, Exercise 13.** In (b) and (d) the vectors \mathbf{b} and \mathbf{b}_i should be nonzero.
- p. 155, Exercise 1.** ... find a product of elementary matrices $E = \cdots E_2 E_1$ so that EA is in echelon form.
- p. 185, Exercise 6a.** nonzero matrix A .
- p. 202, Lemma 2.1.** $Df(\mathbf{a}) = \mathbf{O}$...
- p. 203, Definition.** A critical point \mathbf{a} is a saddle point if for every $\delta > 0$, there are points $\mathbf{x}, \mathbf{y} \in B(\mathbf{a}, \delta)$ with $f(\mathbf{x}) < f(\mathbf{a})$ and $f(\mathbf{y}) > f(\mathbf{a})$.
- p. 207, Exercise 2.** The opposite corner should also be in the first octant, i.e., should have $x, y,$ and z all positive.
- p. 225, Exercise 33.** ... marginal productivity *per dollar* ...
- p. 225, Exercise 34.** On line 2, $Dg(\mathbf{a}) \neq \mathbf{O}$.
- p. 250, footnote.** Kantorovich.
- p. 256, line 6.** Z is a neighborhood of $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{0} \end{bmatrix}$. In Figure 2.4, Z should be slid to the right, containing $V \times \{\mathbf{0}\}$.
- p. 261, Exercise 13a.** Suppose $f \begin{pmatrix} \mathbf{x}_0 \\ t_0 \end{pmatrix} = \frac{\partial f}{\partial t} \begin{pmatrix} \mathbf{x}_0 \\ t_0 \end{pmatrix} = 0$ and the matrix ... is nonsingular. Show that for some $\delta > 0$, there is a \mathcal{C}^1 curve $\mathbf{g}: (t_0 - \delta, t_0 + \delta) \rightarrow \mathbb{R}^2$ with $\mathbf{g}(t_0) = \mathbf{x}_0$ so that ...

p. 271, Proposition 1.6. R' and R'' should overlap in only a “face,” not in a proper subrectangle.

p. 275, Exercise 10. $R \subset \mathbb{R}^n$; line 5 . . . requires at most volume $2A\delta$.

p. 276, Exercise 15b. $D = \{\mathbf{x} \in R : f \text{ is discontinuous at } \mathbf{x}\}$.

p. 316, line –3. Proof of Proposition 5.14.

p. 322, Exercise 10d. The problem should ask only for an example when A and C do not commute. In fact, using the continuity of \det , the astute reader should be able to check that the result of part c *does* hold whenever A and C commute.

p. 326, Proof of Theorem 6.4. In the proof of Theorem 6.4, the reduction to a rectangle is not valid. We have to cover Ω with a union R of rectangles (with rational sidelengths) contained in U . This can then be partitioned into cubes and the proof proceeds.

p. 328, lines 13–15. In the long inequality we should have $\varepsilon \operatorname{vol}(R)(1 + Mn)$ and $\varepsilon \operatorname{vol}(R)(2^n + 2^{n-1}Mn)$. Then let $\beta = \operatorname{vol}(R)(2^n + 2^{n-1}Mn)$.

p. 329, line 1. Section 3, not section 4.

p. 345, lines 4–5. We need the remark here that $\mathbf{g}_2^{-1} \circ \mathbf{g}_1$ is smooth. This can be proved by what should be an exercise in §6.3: Using the notation of part 3 of the Definition on p. 262 of a k -dimensional manifold, perhaps shrinking W , there is a smooth function $\mathbf{h}: W \rightarrow U$ whose restriction to $M \cap W$ is \mathbf{g}^{-1} . (Hint: Without loss of generality, assume $\mathbf{g}(\mathbf{u}_0) = \mathbf{p}$ and write $\mathbf{g}(\mathbf{u}) = \begin{bmatrix} \mathbf{g}_1(\mathbf{u}) \\ \mathbf{g}_2(\mathbf{u}) \end{bmatrix} \in \mathbb{R}^k \times \mathbb{R}^{n-k}$, where $D\mathbf{g}_1(\mathbf{u}_0)$ is nonsingular.)

p. 352, add to Remark: Also, note that we are using the notation $\oint_C \omega$ to denote the integral of ω around the closed curve (or loop) C . This notation is prevalent in physics texts.

p. 355, lines –2 and –1. a should be \mathbf{a} .

p. 368–369, Example 2. In parts a and c, $D = (0, 1) \times (0, 2\pi)$.

p. 380, line 8. Add: “parametrization $\mathbf{g}: U \rightarrow \mathbb{R}^n$ with $U \subset \mathbb{R}_+^k$ and”

p. 381, last line. $\mathbf{g}_i: B(\mathbf{0}, 2) \rightarrow V_i$, and $V_i' = \mathbf{g}_i(B(\mathbf{0}, 1)) \subset V_i$ cover M .

p. 382, line 12. Delete the last equality in the displayed string of equations.

p. 410, lines 4 and 5. All the integrals should be over S^{2m} .

p. 411, Exercise 9. Suppose $U \subset \mathbb{C}$ is open, $f, g: U \rightarrow \mathbb{C}$ are smooth, and $C \subset U$ is a closed curve. Suppose that on C we have $f, g \neq 0$ and $|g - f| < |f|$. Prove that . . .

p. 433, line 5. The 22 entry of $B - I$ should be 2.

p. 444, Example 7, line –3. $\dot{x}_1 = -x_2$.

p. 445, Example 8. Delete the first “the” in the first line.

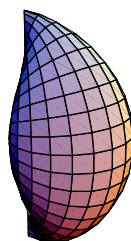
p. 454, Exercise 17c. The result of Exercise 9.2.22 is needed to provide the suggested continuity argument, as well. We should insert a remark that the result of c holds even when the eigenvalues are complex. This is needed for #19.

p. 457, lines 11–12. “Let $W = (\text{Span}(\mathbf{v}_1))^\perp \subset \mathbb{R}^n$ ” should precede the second sentence of the paragraph.

p. 476, #2.2.13. min should be max.

p. 480, #4.5.11a. $DF(\mathbf{x})$ has rank 2 at every point $\mathbf{x} \in M$: Either $x_1 = x_2$ and $x_3 = -x_4$ or $x_1 = -x_2$ and $x_3 = x_4$, so x_1x_2 and x_3x_4 have opposite signs unless they are both 0.

p. 482, #6.2.1: $Dg(\mathbf{f}(\mathbf{x}_0)) = \frac{1}{2(x_0^2 + y_0^2)} \dots$



p. 483, #7.3.12: The picture is not correct.