

**ERRATA** for T. Shifrin's *Multivariable Mathematics: Linear Algebra,  
Multivariable Calculus, and Manifolds*

All of these except those marked with (★) have been corrected in the second printing (June, 2017).

**p. 47, line 11.** In the rightmost determinant, the first entry of the second column should be  $z_1$ .

**p. 79, Exercise 4.**  $x$  should be  $\mathbf{x}$  throughout.

**p. 86, Exercise 12b.** Here  $\mathbf{f}: \mathcal{M}_{n \times n} \rightarrow \mathbb{R}$ .

**p. 91, Example 5.** On the second line it should be  $\mathbf{f}: \mathbb{R}^2 - \{y = 0\} \rightarrow \mathbb{R}^2$ .

**p. 92, Example 7.** The reference should be to Example 3 of Chapter 2, Section 3.

**p. 93, Proposition 2.4, ff.** Standard terminology is that a function  $f$  is  $\mathcal{C}^1$  if  $f$  and its partial derivatives are continuous. Note that in the proof of the Proposition, since the partial derivatives exist, we get continuity of  $f$  along horizontal and vertical lines, which is all we need to apply the Mean Value Theorem. Thus, the Proposition is correct as stated.

**p. 103, Exercise 6.** The symbol for liter (l) looks too much like a 1. For clarity, it would help to change these to  $\ell$ .

**p. 103, Exercise 11.** Prove that a *differentiable* function  $f$  is homogeneous ...

**p. 145, Exercise 13.** In (b) and (d) the vectors  $\mathbf{b}$  and  $\mathbf{b}_i$  should be nonzero.

**p. 155, Exercise 1.** ... find a product of elementary matrices  $E = \cdots E_2 E_1$  so that  $EA$  is in echelon form.

**p. 185, Exercise 6a.** nonzero matrix  $A$ .

**pp. 202, Lemma 2.1.**  $Df(\mathbf{a}) = \mathbf{O}$  ...

(★) **p. 203, lines 8, 13.**  $Df(\mathbf{a}) = \mathbf{O}$ .

**p. 203, Definition.** A critical point  $\mathbf{a}$  is a saddle point if for every  $\delta > 0$ , there are points  $\mathbf{x}, \mathbf{y} \in B(\mathbf{a}, \delta)$  with  $f(\mathbf{x}) < f(\mathbf{a})$  and  $f(\mathbf{y}) > f(\mathbf{a})$ .

**p. 207, Exercise 2.** The opposite corner should also be in the first octant, i.e., should have  $x, y,$  and  $z$  all positive.

**p. 225, Exercise 33.** ... marginal productivity *per dollar* ...

**p. 225, Exercise 34.** On line 2,  $Dg(\mathbf{a}) \neq \mathbf{O}$ .

(★) **p. 233, Lemma 5.1.** Suppose  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  is a basis for  $V$ . Then the equation

$$\mathbf{x} = \sum \text{proj}_{\mathbf{v}_i} \mathbf{x} = \dots$$

holds for all  $\mathbf{x} \in V$  if and only if ...

**p. 250**, footnote. Kantorovich.

**p. 256**, line 6.  $Z$  is a neighborhood of  $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{0} \end{bmatrix}$ . In Figure 2.4,  $Z$  should be slid to the right, containing  $V \times \{\mathbf{0}\}$ .

**p. 261**, Exercise 13a. Suppose  $f\left(\begin{smallmatrix} \mathbf{x}_0 \\ t_0 \end{smallmatrix}\right) = \frac{\partial f}{\partial t}\left(\begin{smallmatrix} \mathbf{x}_0 \\ t_0 \end{smallmatrix}\right) = 0$  and the matrix ... is nonsingular. Show that for some  $\delta > 0$ , there is a  $C^1$  curve  $\mathbf{g}: (t_0 - \delta, t_0 + \delta) \rightarrow \mathbb{R}^2$  with  $\mathbf{g}(t_0) = \mathbf{x}_0$  so that ...

**p. 271**, Proposition 1.6.  $R'$  and  $R''$  should overlap in only a “face,” not in a proper subrectangle.

**p. 275**, Exercise 10.  $R \subset \mathbb{R}^n$ ; line 5 ... requires at most volume  $2A\delta$ .

**p. 276**, Exercise 15b.  $D = \{\mathbf{x} \in R : f \text{ is discontinuous at } \mathbf{x}\}$ .

**p. 316**, line -3. Proof of Proposition 5.14.

**p. 322**, Exercise 10d. The problem should ask only for an example when  $A$  and  $C$  do not commute. In fact, using the continuity of  $\det$ , the astute reader should be able to check that the result of part c *does* hold whenever  $A$  and  $C$  commute.

(★) **p. 326**, Proof of **Theorem 6.4**. In the proof of Theorem 6.4, the reduction to a rectangle is not valid. We have to cover  $\Omega$  with a union  $R$  of rectangles (with rational sidelengths) contained in  $U$ . This can then be partitioned into cubes and the proof proceeds.

**p. 328**, lines 13–15. In the long inequality we should have  $\varepsilon \operatorname{vol}(R)(1 + Mn)$  and  $\varepsilon \operatorname{vol}(R)(2^n + 2^{n-1}Mn)$ . Then let  $\beta = \operatorname{vol}(R)(2^n + 2^{n-1}Mn)$ .

**p. 329**, line 1. Section 3, not section 4.

(★) **p. 345**, lines 4–5. We need the remark here that  $\mathbf{g}_2^{-1} \circ \mathbf{g}_1$  is smooth. This can be proved by what should be an exercise in §6.3: Using the notation of part 3 of the Definition on p. 262 of a  $k$ -dimensional manifold, perhaps shrinking  $W$ , there is a smooth function  $\mathbf{h}: W \rightarrow U$  whose restriction to  $M \cap W$  is  $\mathbf{g}^{-1}$ . (Hint: Without loss of generality, assume  $\mathbf{g}(\mathbf{u}_0) = \mathbf{p}$  and write  $\mathbf{g}(\mathbf{u}) = \begin{bmatrix} \mathbf{g}_1(\mathbf{u}) \\ \mathbf{g}_2(\mathbf{u}) \end{bmatrix} \in \mathbb{R}^k \times \mathbb{R}^{n-k}$ , where  $D\mathbf{g}_1(\mathbf{u}_0)$  is nonsingular.)

**p. 352**, add to **Remark**: Also, note that we are using the notation  $\oint_C \omega$  to denote the integral of  $\omega$  around the closed curve (or loop)  $C$ . This notation is prevalent in physics texts.

**p. 355**, lines -2 and -1.  $a$  should be  $\mathbf{a}$ .

**p. 368–369**, Example 2. In parts a and c,  $D = (0, 1) \times (0, 2\pi)$ .

**p. 380**, line 8. Add: “parametrization  $\mathbf{g}: U \rightarrow \mathbb{R}^n$  with  $U \subset \mathbb{R}_+^k$  and”

**p. 381**, last line.  $\mathbf{g}_i: B(\mathbf{0}, 2) \rightarrow V_i$ , and  $V'_i = \mathbf{g}_i(B(\mathbf{0}, 1)) \subset V_i$  cover  $M$ .

**p. 382**, line 12. Delete the last equality in the displayed string of equations.

**p. 410**, lines 4 and 5. All the integrals should be over  $S^{2m}$ .

**p. 411**, Exercise 9. Suppose  $U \subset \mathbb{C}$  is open,  $f, g: U \rightarrow \mathbb{C}$  are smooth, and  $C \subset U$  is a closed curve. Suppose that on  $C$  we have  $f, g \neq 0$  and  $|g - f| < |f|$ . Prove that ...

**p. 433**, line 5. The 22 entry of  $B - I$  should be 2.

**p. 444**, Example 7, line -3.  $\dot{x}_1 = -x_2$ .

**p. 445**, Example 8. Delete the first “the” in the first line.

(★) **p. 454**, Exercise 17c. The result of Exercise 9.2.22 is needed to provide the suggested continuity argument, as well. We should insert a remark that the result of c holds even when the eigenvalues are complex. This is needed for #19.

**p. 457**, lines 11–12. “Let  $W = (\text{Span}(\mathbf{v}_1))^\perp \subset \mathbb{R}^n$ ” should precede the second sentence of the paragraph.

**p. 476**, #2.2.13. min should be max.

**p. 480**, #4.5.11a.  $DF(\mathbf{x})$  has rank 2 at every point  $\mathbf{x} \in M$ : Either  $x_1 = x_2$  and  $x_3 = -x_4$  or  $x_1 = -x_2$  and  $x_3 = x_4$ , so  $x_1x_2$  and  $x_3x_4$  have opposite signs unless they are both 0.

**p. 482**, #6.2.1:  $Dg(\mathbf{f}(\mathbf{x}_0)) = \frac{1}{2(x_0^2 + y_0^2)} \dots$

**p. 483**, #7.3.12: The picture is not correct.

