

# ABSTRACT ALGEBRA: A Geometric Approach

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## Errata and Typographical Errors

**p. 17, lines 3-4.**  $N$  is the smallest positive integer having . . . Note, in particular, that  $N$  cannot be prime; now, suppose . . .

**p. 18, Exercise 3.**  $\gcd(0, m) = |m|$ .

**p. 18, Exercise 13.**  $\mu = |ab|$  and  $|ab|/d$ .

**p. 21, line 4.** Delete “)”.

**p. 24, line 8.** Delete the second “ $p$ .”

**p. 35, Exercise 38.** Using  $e = 101$ , check that  $d = 317$ . In the first line of the table, 1663 should be 1633. . . Using the same  $N$  and  $e = 1601$ , . . .

**p. 39, line 14.** . . .  $a^{-1}(ab) = a^{-1} \cdot 0 = 0$ . . .

**p. 47, line 10.** Given  $n \in \mathbb{Z}$ , exactly one of the statements

$$n \in \mathbb{Z}^+, n = 0, \text{ or } -n \in \mathbb{Z}^+$$

is true. (This is called the trichotomy principle.)

line 15.  $\mathbb{Z}^+$  should be  $\mathbb{Z}^+$ .

line 21. Given  $x \in F$ , exactly one of the statements

$$x \in F^+, x = 0, \text{ or } -x \in F^+$$

is true.

**p. 66, Exercise 14.** This is correct when  $v \geq 0$ . What's the answer when  $v < 0$ ?

**p. 66, Exercise 17.** Delete the first  $dx$ .

**p. 91, lines 1-4.**  $p_1(x), \dots, p_m(x)$  should be *distinct* irreducible polynomials. And  $\deg(r_j(x)) < \deg(p_j(x)^{v_j})$  for  $j = 1, \dots, m$ .

**p. 92, Exercise 9.** Parts a. and b. are much harder than I thought when I wrote the problem. Even deciding if there are any irreducible elements of  $\mathbb{Z}_6[x]$  seems quite difficult.

**p. 116, line 3.**  $(ya)$  should be  $(ya^{-1})$  at the end of the line.

**p. 117, Example 3, last line.**  $\langle 2 \rangle = \langle 4 \rangle \subset \mathbb{Z}_6$ , and  $4 = 2 \cdot 2$ , so 4 is a multiple of 2 by a zero-divisor. On the other hand,  $4 = -2$ , so 4 is *also* a multiple of 2 by a unit! In fact, in  $\mathbb{Z}_n$  it can be shown that  $\langle a \rangle = \langle b \rangle \iff a = sb$  for some unit  $s$ . (**N.B.** One can construct a commutative ring  $R$  having elements  $a, b$  satisfying  $\langle a \rangle = \langle b \rangle$  and yet  $a = sb$  for *no* unit  $s$ !)

**p. 118, line -10.** Proposition 2.2 of Chapter 3.

**pp. 123-4, Exercise 16.** A few symbols didn't print here:

a. Given an ideal  $J \subset S$ , define

$$\mathcal{J} = \{a \in R : \phi(a) \in J\} \subset R.$$

(This is usually denoted by  $\phi^{-1}(J)$ .) Prove that  $\mathcal{J}$  is an ideal.

b. Given an ideal  $I \subset R$ , define

$$\mathcal{J} = \{\phi(a) : a \in I\} \subset S.$$

... Prove that  $\mathcal{J}$  is an ideal, provided  $\phi$  maps onto  $S$ . ...

**p. 128**, last line:  $\bar{3}$  should be 3.

**p. 130**, line 3 of the proof of Theorem 2.4.  $\text{ev}_\alpha$  maps  $F[x]$  onto  $F[\alpha]$  and has ...

**p. 132**, line 5 of the proof of Corollary 2.6. The only polynomial  $f(x) \in \mathbb{Q}[x]$  having  $c$  as a root ...

**p. 133**, **Remark** and **Corollary 2.8**. In the displayed statement marked with (\*), we must assume that  $\alpha \notin \mathbb{Q}$ . In the application, we should point out that  $\alpha \notin \mathbb{Q}$  (e.g., by applying the result of Exercise 2.2.11a).

**p. 138**, **Exercise 28a**. We must assume  $p \neq 0$ .

**p. 143**, line 8.  $\psi(a + bi) = a + 3b \pmod{5}$ .

**p. 163**, line 4. for some nonnegative integer  $r$  ...

**p. 184**, lines 9–10. ... in  $\mathbb{Z}_5^\times$  there is only one.

**p. 194**, lines 3–4. That is, the conjugate of  $F = F_1$  (which is the flip fixing vertex 1) by  $R$  is  $F_2$  (which is the flip fixing vertex 2).

**p. 258**, line 8 of Proof of Theorem 5.8.  $k(aH) \subset aH$ .

**p. 260**, **Exercise 18**. Assume also that if  $\alpha = 1$ , then  $m > 1$ .

**p. 265**, **last line** "... extending  $\phi(\dots)$  and carrying  $\alpha$  to  $\alpha'$ ."

**p. 266**, line 10. "If  $\tilde{\phi}(p(\alpha)) = 0, \dots$ "

Proof of Proposition 6.4. Assume  $K \neq F$ .

**p. 268**, line 4.  $F'[\alpha_j]$  should be  $F'[\alpha'_j]$ .

**p. 269**, line 8. "splits in  $K$ ."

**p. 272**, line 15.  $-\omega$  should be  $\bar{\omega}$ .

**p. 275**, line 8.  $t$  should be  $\ell$ .

**p. 277**, **Exercise 5**.  $\phi\psi^2$  should be  $\psi\phi^2$ .

**p. 287**, line -8.  $|x_1y_2 - x_2y_2|$  should be  $|x_1y_2 - x_2y_1|$ .

**p. 302**, line 1.  $\{\infty\}$  should be  $\mathbb{P}_\infty^1$ .

**p. 304**, last line. The coordinates of  $B$  and  $C$  are reversed.

**p. 308**, **Figure 11**.  $\alpha, \beta, \gamma$  should be  $\beta, \gamma, \alpha$ , respectively.

**p. 310–11**.  $t' = g(t) = -\frac{d+et}{a+bt}$ , which makes the matrix  $\begin{bmatrix} a & b \\ -d & -e \end{bmatrix}$  and  $g^{-1}(t) = -\frac{d+at}{e+bt}$ .

**p. 316, Exercise 18.** This is false with the definitions provided. Indeed, we should really say that *six points are in general position provided no three are collinear and they do not all lie on a conic*. Then Exercise 18 becomes a triviality.

**p. 319, line 6.**  $\mathbf{x}_1, \dots, \mathbf{x}_5 \in \mathbb{R}^3$ .

**p. 324, line 4.**  $\mathbf{v}_2 = \frac{1}{3\sqrt{2}}(1, -4, 1)$ .

**p. 324, line -5.**  $y_i = \sqrt{|\lambda_i|}x_i$ .

**p. 325, line 11.**  $z_i = \sqrt{|\lambda_i|}y_i$ .

**p. 342, Exercise 20.** A few symbols didn't print here:

... Associate to the line  $\overline{PQ}$  the point  $\mathcal{A} = [a_{01}, a_{02}, a_{03}, a_{12}, a_{13}, a_{23}] \in \mathbb{P}^5$ .

a. Prove that  $\mathcal{A}$  is a well-defined point in  $\mathbb{P}^5$  determined only by the line. (... the point  $\mathcal{A} \in \mathbb{P}^5$  doesn't change. ...  $a'_{ij} = p'_i q'_j - p'_j q'_i$ , we have  $\mathcal{A}' = \mathcal{A} \in \mathbb{P}^5$ .)

b. Show that  $\mathcal{A}$  satisfies the (homogeneous) quadratic equation ...

c. Given two lines  $\ell$  and  $m$ , let  $\mathcal{A}$  and  $\mathcal{B}$  be the corresponding points in  $\mathbb{P}^5$ . ...

**p. 361, line 15.** "two" should be "four."

**p. 363, Exercise 6.** The reference should be to Lemma 5.7.

**p. 367, Exercise 30b,c.** The factors of  $\frac{1}{2}$  should be 2's.

**p. 371, line 6.**  $Q$  should be  $\mathbb{Q}$ .

**p. 378, Exercise 5.**  $h = g \circ f$ ; interchange  $f$  and  $g$  in parts a., b., c.

**p. 387, Figure 1.** The labels  $P$  and  $P^{-1}$  should be interchanged.

**p. 401, Exercise 1a.** ... prove  $\lambda^n$  is an eigenvalue of  $T^n$  ....