MATH 3300 - Final

December 10 2018

Name: ______________________________

- No notes, calculators, other electronic devices are allowed.
- Full credit may not be given if sufficient justification is not provided.
- Academic Honesty Student Honor Code: "I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others."
- This exam has 11 pages. You must check that no page is missing.

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Question 1. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ z - y \\ x \end{bmatrix}.$$

1. (2 points) Find the matrix A of T.

2. (2 points) Find the reduced echelon form of A. Show all your work.

3. (3 points) Is T one-to-one? Briefly justify your answer.

4. (3 points) Is T onto? Briefly justify your answer.

Question 2. (10 points) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix}$.

1. (6 points) Find the inverse A^{-1} of A.

2. (4 points) Let $B = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$. Find the 3×3 matrix X that satisfies the matrix equation $XA = B$. Show all your work.

Question 3. (10 points) Find the LU factorization of $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 9 \\ 6 & 8 & 19 \end{bmatrix}$. Show all you work.

Question 4. (10 points) Let $A = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 1 & 3 \\ 2 & 4 & 17 & 5 \end{bmatrix}$.

1. (3 points) Find the reduced echelon form (columns with leading 1s must have 0s everywhere else) of A. Show all your work.

2. (2 points) Find a basis for the column space of A.

3. (2 points) Find a basis for the null space of A.

4. (1 point) What is rank(A)?

Question 5. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 2 & 2 & 3 \end{bmatrix}$.

1. (4 points) Find $\det(A)$. Show your work.

2. (2 points) Is A invertible? Briefly justify your answer.

3. (2 points) Is $\dim(\text{Row}(A)) = 3$? Briefly justify your answer.

Question 6. (10 points) Let $W = \text{Span}\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \\ 0 \end{bmatrix} \right\}$. Find an orthogonal basis for W by using the Gram-Schmidt process.

Question 7. (10 points) Find the least-squares solution and the least squares error of the system

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Question 8. (10 points) Diagonalize $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

Question 9. (10 points) Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$. Find the singular value decomposition of A.

Question 10. (10 points) Fill in the blanks.

1. If $A = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$, then

   $A^4 =$

2. If A and B are 2×2 matrices such that $\det(A) = -1$ and $\det(B) = 2$, then

   $\det(2A^3B^{-1}) =$

3. If $\mathbf{v}_1$ is an eigenvector of A with eigenvalue -2 and $\mathbf{v}_2$ is an eigenvector with eigenvalue 6, then

   $A(2\mathbf{v}_1 - \mathbf{v}_2) =$

4. If $\mathbf{u}$ is a vector in $\mathbb{R}^3$ and W is a subspace of $\mathbb{R}^3$ that is orthogonal to $\mathbf{u}$, then

   $\text{proj}_W(\mathbf{u}) =$

5. Let $W = \text{Span}\{\mathbf{e_1} + \mathbf{e_2}\}$ (the line $y = x$) in $\mathbb{R}^2$. Find its orthogonal complement.

   $W^\perp =$