MATH 3300 - EXAM 2 PRACTICE

Name: ______________________________

- No notes, calculators, other electronic devices are allowed.
- Full credit may not be given if sufficient justification is not provided.
- Academic Honesty Student Honor Code: "I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others."
- This exam has 9 pages. You must check that no page is missing.

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Total	

Question 1. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$.

1. (3 points) Find the reduced echelon form (columns with leading 1s must have 0s everywhere else) of A. Show all your work.

2. (3 points) Find a basis for the column space of A.

3. (3 points) Find a basis for the row space of A.

4. (1 point) Find $\dim(\text{Nul}(A))$.

Question 2. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 5 & 0 & 7 \\ 6 & 7 & 1 & 9 \\ 1 & -1 & 0 & 2 \end{bmatrix}$.

1. (5 points) Find $\det(A)$. Show all your work.

2. (2 points) Is A invertible? Briefly justify your answer.

3. (3 points) Find $\det(-A^T A^3)$. Show all your work.

Question 3. (10 points) Let $A = \begin{bmatrix} 10 & -12 & 8 \\ 0 & 1 & 0 \\ -8 & 12 & -6 \end{bmatrix}$.

1. (4 points) Find the eigenvalues of A.

2. (5 points) Find a basis for the eigenspace of each eigenvalue and state its dimension.

3. (1 point) Is A diagonalizable? Explain your answer.

Question 4. (10 points) Let $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. Diagonalize A and use your answer to find A^n for all integer n.

Question 5. (10 points) Fill in the blanks or circle Truth of False.

1. The rows of an $n \times n$ matrix of rank n span $\mathbb{R}^n$. **T** or **F**.

2. If A is an $m \times n$ matrix, then $\dim(\text{Nul}(A)) + \text{rank}(A) =$ _____

3. A square matrix with a row consisting of zeroes is not invertible. **T** or **F**.

4. $\det \begin{bmatrix} \mathbf{r}_1 \\ 2\mathbf{r}_2 - \mathbf{r}_1 \end{bmatrix} =$ ____ $\det \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{bmatrix}$. Here $\mathbf{r}_1$ and $\mathbf{r}_2$ are row vectors in $\mathbb{R}^2$.

5. $\det(A + 2B) = \det(A) + 2\det(B)$ for all square matrices A and B. **T** or **F**.

6. A matrix has a unique eigenvector for each eigenvalue. **T** or **F**.

7. $\det(P^{-1}AP) = \det(A)$.

8. An $n \times n$ matrix with n distinct eigenvalues is diagonalizable. **T** or **F**.

9. There exists an $n \times n$ matrix with $n + 1$ distinct eigenvalues. **T** or **F**.

10. If $\mathbf{v}$ is an eigenvector of A with eigenvalue λ, then it is an eigenvalue of A^2 with eigenvalue λ^2. **T** or **F**.