

MATH 3300 - EXAM 1 PRACTICE

Name: _____

- No notes, calculators, other electronic devices are allowed.
- Full credit may not be given if sufficient justification is not provided.
- Academic Honesty Student Honor Code: “I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others.”
- This exam has 5 pages. You must check that no page is missing.

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Question 1. (10 points) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$.

1. (4 points) Find the reduced echelon form (columns with leading 1s must have 0s everywhere else) of A . Show all your work.

2. (3 points) Are the columns of A linearly independent? Briefly justify your answer.

3. (3 points) Do the columns of A span \mathbb{R}^3 ? Briefly justify your answer.

Question 2. (10 points) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + y + z \\ x - z \end{bmatrix}.$$

1. (2 points) Find the matrix A of T .
2. (2 points) Find the reduced echelon form of A . Show all your work.
3. (3 points) Is T one-to-one? Briefly justify your answer.
4. (3 points) Is T onto? Briefly justify your answer.

Question 3. (10 points) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. Show all your work.

1. (7 points) Find the inverse A^{-1} of A .

2. (3 points) Solve the linear system of equations $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Show all your work.

Question 4. (10 points) Find the LU factorization of $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 3 \\ -1 & 2 & 1 \end{bmatrix}$.