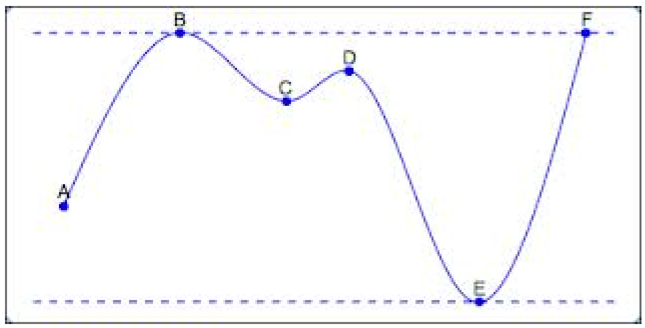
Section 4.3: Monotonic Functions and the First Derivative Test

We start by recapping vocabulary words from Section 4.1:
Local Extrema: A local minimum of a function f occurs at a point $x = c$ if $f(c)$ is smaller than any other point nearby. Similarly, a local maximum occurs at a point $x = c$ if $f(c)$ is larger than any other point nearby.

Global Extrema: A global or absolute maximum of a function f occurs at a point $x = c$ if, for all x in the domain of f, $f(x) \leq f(c)$. Similarly, you have an absolute minimum at $x = c$ if $f(x) \geq f(c)$.

Example: Determine which of the six points below are local maxima, local minima, absolute maxima, and absolute minima



Monotonic Functions:
A function that is increasing or decreasing on an interval I is called **monotonic on** I. For functions that are continuous and differentiable, $f'(x)$ will dictate the behavior of $f(x)$. Remember that from Section 3.4, a positive velocity means that the object is rising, whereas a negative velocity means that the object is falling. This ultimately translates to:

If $f'(x)$ is positive on an interval I, then $f(x)$ is monotonically increasing on the same interval.

If $f'(x)$ is positive on an interval I, then $f(x)$ is monotonically increasing on the same interval.

If we can find where $f'(x)$ is zero (or undefined), we can break up the domain of $f(x)$ into monotonic components. Remember that such points are called critical points.

Example: Determine where the following function is increasing/decreasing: $f(x) = x^3 - 12x - 5$.

This then leads us to the First Derivative Test:

> Suppose that c is a critical point of a continuous function $f(x)$ and that $f(x)$ is differentiable at every point in some interval containing c, except possibly at c itself. Moving across c from left to right, then:
>
> 1. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a local minimum.
> 2. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a local maximum.
> 3. If no sign change happens at c, then $f(c)$ is not a local extremum.

Examples: For the following, determine for which intervals the function $f(x)$ is increasing/decreasing and label all critical points as either local min, local max, or not an extremum.

(a) $f(x) = -3x^2 + 9x + 5$ (b) $g(x) = (x + 1)^{1/3}(x - 9)$ (c) $h(x) = x^2\sqrt{5 - x}$

(d) $j(x) = \mathbf{e}^x(x^2 - 3)$ (e) $k(x) = x^2 \ln(x)$