Section 2.6: Limits Involving Infinity.

We again use the graphical approach to consider limits. In this case, let $f(x) = \dfrac{1}{x}$. Answer the following:

$$\lim_{x\to-\infty} \frac{1}{x} = \qquad \lim_{x\to\infty} \frac{1}{x} = \qquad \lim_{x\to0^-} \frac{1}{x} = \qquad \lim_{x\to0^+} \frac{1}{x} =$$

Definition: A **Horizontal Asymptote** of the graph of a function $f(x)$ is the horizontal line $y = b$, where either:

$$\lim_{x\to-\infty} f(x) = b \qquad \text{or} \qquad \lim_{x\to\infty} f(x) = b$$

Definition: The line $x = a$ is a **vertical asymptote** of a function $f(x)$ if either:

$$\lim_{x\to a^+} f(x) = \pm\infty \qquad \text{or} \qquad \lim_{x\to a^-} f(x) = \pm\infty$$

Examples: Rational Functions and Horizontal Asymptotes.
Let $f(x) = 5x^3 + 9x + 27$, $g(x) = 4x^2 - 5$, and $h(x) = 8x^3 + 4x^2 - 3x + 9$. Determine the horizontal asymptote (if any) of the following functions:

(a) $\left(\dfrac{f}{g}\right)(x)$ (b) $\left(\dfrac{g}{f}\right)(x)$ (c) $\left(\dfrac{f}{h}\right)(x)$

Examples: Rational Functions and Vertical Asymptotes.
Determine whether or not the following functions have a vertical asymptote at $x = 3$.

(a) $f(x) = \dfrac{5}{x-3}$ (b) $g(x) = \dfrac{x^2-9}{x-3}$ (c) $h(x) = \dfrac{x^2+5x+1}{x^2-4x+3}$

Examples: Non-Rational Functions and Asymptotes.
Use your Pre-Calculus knowledge of graphs, as well as the formal definitions mentioned at the top of this worksheet, to determine the asymptote(s) of the following functions:

(a) $y = e^x$ (b) $y = \ln(x)$ (c) $y = \tan(x)$

(d) $y = \csc(x)$ (e) $y = \arctan(x)$

Slant Asymptotes.
A **slant asymptote** exists when the degree of the numerator is *exactly* one bigger than the degree of the denominator. See the examples below to learn how to compute slant asymptotes:

Examples: Compute all asymptotes of the following rational functions:

(a) $f(x) = \dfrac{5x^2+9x+1}{x^2+1}$ (b) $g(x) = \dfrac{6x^3+6x+1}{x^4-16}$ (c) $h(x) = \dfrac{6x^2+10x+10}{x-2}$

(d) $j(x) = \dfrac{5x^3+2x+1}{x^2-9}$ (e) $k(x) = \dfrac{-4x^5+1}{x^2-6x+8}$