Section 2.5B: More on Continuity.

Review: Answer the following.

(1) Let $h(t) = \dfrac{t^2 + 3t - 18}{t - 3}$.

Create a continuous extension of $h(t)$.

(2) Let $f(x)$ be defined piecewise as below.

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 0 \\ 5ax & \text{for } x \leq 0 \end{cases}$$

For what value(s) of a will result in $f(x)$ being continuous everywhere?

Examples and Properties of Continuous Functions:

1. All polynomials are continuous everywhere.
2. Rational functions are continuous everywhere in its domain.
3. The functions $\sin(x)$ and $\cos(x)$ are continuous everywhere.
4. The functions $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ are continuous for all x, except for their respective vertical asymptotes.
5. The exponential function b^x (with b a positive real number) is continuous everywhere.
6. The logarithmic function $\log_b(x)$ (with b a positive real number) is continuous on the interval $(0, \infty)$.
7. The root function $f(x) = \sqrt[n]{x}$ is continuous everywhere if n is an odd whole number.
8. The root function $f(x) = \sqrt[n]{x}$ is continuous at $[0, \infty)$ if n is an even whole number.
9. Adding, subtracting, and multiplying continuous functions will result in a continuous function.
10. Dividing two continuous functions will result in a continuous function as long as the denominator is not zero.
11. The composition of two continuous functions will result in a continuous function.

Examples: Use the properties listed above to explain why the following functions are also continuous everywhere.

$$f(x) = 8^{x^2+4x} \qquad h(x) = \sin(4x - 21)$$

$$g(x) = x^2 \cos(e^x) \qquad j(x) = \frac{\sqrt[5]{x}}{x^2 + 9}$$

A function $f(x)$ is said to be continuous on a closed interval $[a, b]$ if the following criteria are met:

- $f(x)$ is continuous from the right at $x = a$.
- $f(x)$ is continuous for all x in (a, b).
- $f(x)$ is continuous from the left at $x = b$.

This then brings us to the first major theorem of the semester: **Intermediate Value Theorem**.

If a function $f(x)$ is continuous on a closed interval $[a, b]$, and if a value $y = c$ is between $f(b)$ and $f(a)$, then there exists at least one value $x = d$ in $[a, b]$ such that $c = f(d)$.

An application of the Intermediate Value Theorem (IVT) is determining whether an equation has a solution.

Examples: Answer the following.

1. Use the IVT to show that the equation $\cos(3x) = 6x - 2$ has a solution on the interval $\left[-\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$.

2. Use the IVT to show that the equation $x^3 = 10x - 4x$ has three distinct solutions.