Section 2.1: Average and Instantaneous Rate of Change.

The *average rate of change* of a function $y = f(x)$ over an interval $[a, b]$ is:

$$\frac{f(b) - f(a)}{b - a}.$$

This formula should look familiar. This is the slope of a line through two points.

Now let $h = b - a$ and $a + h = b$. We can then transform this formula to:

$$\frac{f(b) - f(a)}{h}.$$

As b gets closer and closer to a, then h gets closer and closer to zero. This then gives us the concept of an **instantaneous rate of change**, which is computed as a limit:

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Examples:

1. Compute an equation of the line containing points $(-2, 3)$ and $(5, 9)$.

2. Let $f(x) = 9 + \sqrt{x - 5}$. Compute the average rate of change of $f(x)$ on the interval [6, 21]. Then determine the instantaneous rate of change of $f(x)$ at $x = 6$.

Section 3.1/3.2: Tangent Lines and the Derivative.
The **tangent line** of $f(x)$ at a point $x = a$ is the linear function: $y = m(x - a) + f(a)$. The value m is

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided that this limit exists. If $m = 0$, then we have what's called a horizontal tangent line.

The **derivative** of a function $f(x)$ with respect to the variable x is the function $f'(x)$ such that:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Note that when computing the tangent line at $x = a$, your slope $m = f'(a)$.

Examples:

1. Compute the tangent line of $f(x) = \frac{5}{x}$ at $x = 2$.

2. Determine the derivative of $f(x) = 2x^2 + 5$. Where will $f(x)$ have horizontal tangents?

3. Determine the derivative of $f(x) = \frac{1}{\sqrt{2x + 4}}$. Where will $f(x)$ have horizontal tangents?