NAME: ____ GROUP WORK #6

PART 1: Conceptual Questions. Select the most correct answer amongst the answer choices listed below.

_______ 1. Suppose that $f(x)$ has a critical point at $x = a$. Which of the following must be true?

A. $f'(a)$ must be positive.
B. $f'(a)$ must be negative.
C. $f'(a)$ must equal zero.
D. $f'(a)$ must be undefined.
E. None of the above.

_______ 2. Which of the following functions **will not** have an absolute maximum on its given interval?

A. $f(x) = e^x$ on the interval $(-\infty, 5]$.
B. $g(x) = x^2 + 9$ on the interval $[-3, 3]$.
C. $h(x) = 1/x$ on the interval $[-2, 2]$.
D. $j(x) = \sec(x)$ on the interval $[-1, 1]$.
E. All the functions above will attain an absolute max in their given intervals.

Part 2: Examples. Give an explicit example (not a drawing) of a function that satisfies the following conditions:

3. A function $f(x)$ that is continuous at $x = 25$, but is not differentiable at $x = 25$.

One function that works is $f(x) = \sqrt[3]{x - 25}$.

Cube roots are continuous everywhere, but $f'(x) = \dfrac{1}{\sqrt[3]{(x-25)^2}}$, which is undefined at $x = 25$.

4. A function $g(x)$ that has infinitely many inflection points.

Two functions should come to mind: $g(x) = \sin(x)$ and $g(x) = \cos(x)$. Two other functions also work: $\tan(x)$ and $\cot(x)$. They change concavity at the midpoints of their asymptotes. Two functions that do not work are $\sec(x)$ and $\csc(x)$: These two functions alternate concave up/down, but the concavity changes at the asymptotes, which are not points on the graph.

5. A function $h(x)$ that evaluates to an indeterminate form at $x = 2$.

There are four indeterminate forms: $\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$, $\infty - \infty$, and $0 \cdot \infty$. An example of each are:

$h(x) = \dfrac{x-2}{x-2}$, $h(x) = \dfrac{\ln(x-2)}{\ln(x-2)}$, $h(x) = \sin(x-2) \cdot \dfrac{1}{x-2}$, and $h(x) = \dfrac{1}{x-2} - \dfrac{x}{x-2}$.

6. A function $j(x)$ that evaluates to an indeterminate power at $x = 3$.

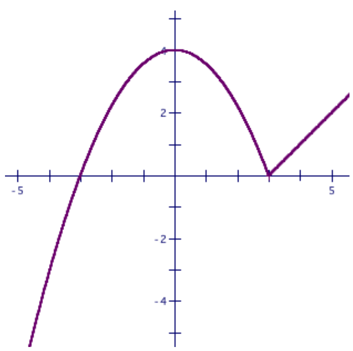
There are three indeterminate powers: 0^0, 1^∞, and ∞^0. An example of each are:

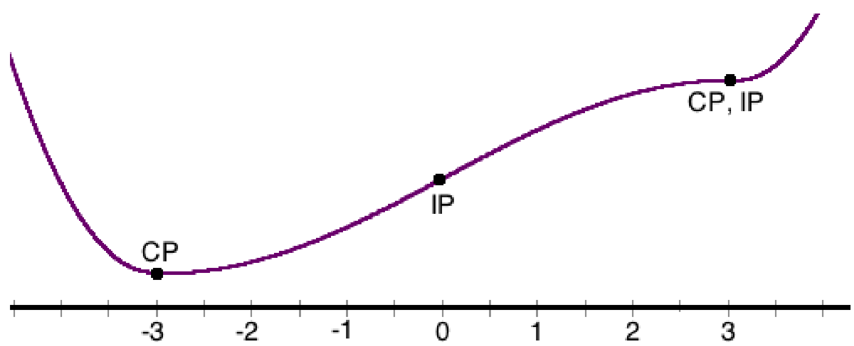
$j(x) = (x-3)^{x-3}$, $j(x) = (x-2)^{1/(x-3)}$, and $j(x) = \left(\dfrac{1}{x-3}\right)^{x-3}$.

7. Suppose that a function $f(x)$ is twice differentiable on an interval I. Fill in the following chart with one of the following choices: positive, negative, increasing, decreasing, concave up, concave down, or none of the above.

	Behavior of $f(x)$	Behavior of $f'(x)$	Behavior of $f''(x)$
$f(x)$ is always positive on I	positive	none	none
$f(x)$ is always negative on I	negative	none	none
$f'(x)$ is always positive on I	increasing	positive	none
$f'(x)$ is always negative on I	decreasing	negative	none
$f''(x)$ is always positive on I	concave up	increasing	positive
$f''(x)$ is always negative on I	concave down	decreasing	negative

8. You are given the graph of $f'(x)$ (**the derivative**) below. You may assume that $f(x)$ is continuous everywhere and is twice differentiable everywhere.





Use the graph (as well as the middle column of the chart in the front) to answer the following. When appropriate, give your answers in interval notation.

(a) $f'(x)$ is positive at $(-3,3) \cup (3,\infty)$ and negative at $-\infty,-3)$.

(b) $f(x)$ is increasing at $(-3,3) \cup (3,\infty)$ and decreasing at $(-\infty,-3)$.

(c) $f(x)$ is concave up at $(-\infty,0) \cup (3,\infty)$ and concave down at $(0,3)$.

(d) $f(x)$ has critical point(s) at $x = -3, 3$ and inflection point(s) at $x = 0, 3$.

(e) $f(x)$ has an absolute min at $x = -3$ and an absolute max at $x =$ none.

(f) Use all the given information to give a rough sketch of the graph of $f(x)$.

Using parts (b) and (c), you'll have:

x			-3			0			3		
$f'(x)$	$-$	$-$	|	$+$	$+$	$+$	$+$	$+$	|	$+$	$+$
$f''(x)$	$+$	$+$	$+$	$+$	$+$	|	$-$	$-$	|	$+$	$+$
$f(x)$	DCU		|	ICU		|	ICD		|	ICU	