NAME: ____ **Group Work 6**

1. From 4.1, Theorem #3, we discussed that if a function $f(x)$ is continuous on a closed interval, then it is guaranteed to attain a maximum and a minimum value in that interval, and these extrema will occur either at a critical point or at an end point.

While these two criteria are sufficient to guarantee extrema, they are not necessary. Answer the following to illustrate this fact.

a. Using the function $f(x) = \sin(x)$, determine an **open** interval (a, b) in which $f(x)$ attains its maximum and minimum values in your interval.

Interval: $(0, 2\pi)$ Min $= -1$ at $x = \frac{3\pi}{2}$ Max $= 1$ at $x = \frac{\pi}{2}$

b. Create a function $g(x)$ that is **not continuous** at $x = 1$, but $g(x)$ attains both a maximum and a minimum on the interval $[-4, 4]$. Sketch the graph below.

Lots of functions work here. An example of one is the floor function $g(x) = \lfloor x \rfloor$. It is not continuous at $x = 1$, but attains a maximum and minimum value in the interval.

This means that if $f(x)$ is continuous on a closed interval $[a, b]$, it is guaranteed to have both an absolute max and absolute min in the interval. If either of those criteria fails, you are not guaranteed anything, but it can still have extrema.

2. Determine the maximum and minimum values of the following functions for the given intervals:

a. $f(x) = \frac{1}{x} + \ln(x)$ for $0.5 \leq x \leq 4$.

From continuity rules, this function is continuous for all values $x > 0$. Thus it is continuous in the given interval. Therefore the Extreme Value Theorem applies. $g'(x) = -\frac{1}{x^2} + \frac{1}{x}$. This is undefined at $x = 0$ (outside the given domain). This is equal to zero at $x = 1$. Checking CPs and EPs, you'll get $g(1) = 1$, $g(0.5) \approx 1.30685$, and $g(4) \approx 1.6463$. The absolute maximum is $\frac{1}{4} + \ln(4)$ and the absolute minimum is 1.

b. $g(x) = 2 - |x|$ for $-1 \leq x \leq 3$. Hint: Sketch your graph.

If you sketch this, you'll see that your graph has one critical point at $x = 0$. Since this is a piecewise polynomial, it is continuous everywhere. Thus the Extreme Value Theorem applies. Using CPs and EPs, you'll get $f(-1) = 1$, $f(0) = 2$, and $f(3) = -1$. Thus, the absolute minimum is -1 and the absolute maximum is 2.

3. Compute the following limits:

(a)

$$\lim_{x\to 1} (4x-3)^{1/(1-x)} = L$$

This evaluates to 1^∞, which is an indeterminate power.

$$\lim_{x\to 1} \ln\left((4x-3)^{1/(1-x)}\right) = \ln(L)$$

$$\lim_{x\to 1} \frac{\ln(4x-3)}{1-x} = \ln(L)$$

This evaluates to zero over zero, so L'Hopital's rule applies.

$$\lim_{x\to 1} \frac{\frac{4}{4x-3}}{-1} = \ln(L)$$

$$\lim_{x\to 1} -\frac{4}{4x-3} = \ln(L)$$

$$-4 = \ln(L)$$

Now exponentiate both sides.

$$e^{-4} = L$$

(b)

$$\lim_{x\to\infty} (4x-3)^{1/(1-x)} = L$$ This evaluates to ∞^0, which is an indeterminate power.

$$\lim_{x\to\infty} \ln\left((4x-3)^{1/(1-x)}\right) = \ln(L)$$

$$\lim_{x\to\infty} \frac{\ln(4x-3)}{1-x} = \ln(L)$$ This evaluates to ∞ over ∞, so L'Hopital's rule applies.

$$\lim_{x\to\infty} \frac{\frac{4}{4x-3}}{-1} = \ln(L)$$

$$\lim_{x\to\infty} -\frac{4}{4x-3} = \ln(L)$$ As $x \to \infty$, the denominator gets infinitely large, which means the fraction goes to zero.

$$0 = \ln(L)$$ Now exponentiate both sides.

$$e^0 = 1 = L$$

4. Let $f'(x) = (x-4)^2(x+3)(x-1)$. Assuming that $f(x)$ has domain $(-\infty, \infty)$, determine the intervals for which $f(x)$ is increasing and which intervals $f(x)$ is decreasing.