NAME: ____________________ **Group Work 5**

1. Compute the derivative of the following functions:

(a) $y = (e^x)^2$ $\quad y = e^{2x}$

$y = e^u$, with $u = 2x$

$$\frac{dy}{dx} = e^{2x} \cdot 2$$

(b) $y = e^{(x^2)}$

$y = e^u$ with $u = x^2$

$$\frac{dy}{dx} = e^{x^2} \cdot 2x$$

2. Determine the equation of the tangent line of the equation below at the point $(2, 1)$:

$$4xy^3 + 3xy = 14y$$

$4xy^3 + 3xy$	$=$	$14y$	Use implicit. Apply product rule as necessary.
$4y^3 + 4x(3y^2)\frac{dy}{dx} + 3y + 3x\frac{dy}{dx}$	$=$	$14\frac{dy}{dx}$	Gather $\frac{dy}{dx}$ terms to one side.
$12xy^2\frac{dy}{dx} + 3x\frac{dy}{dx} - 14\frac{dy}{dx}$	$=$	$-4y^3 - 3y$	Factor out $\frac{dy}{dx}$.
$\left(12xy^2 + 3x - 14\right)\frac{dy}{dx}$	$=$	$-4y^3 - 3y$	Divide.
$\frac{dy}{dx}$	$=$	$\frac{-4y^3 - 3y}{12xy^2 + 3x - 14}$	Now, plug in (2, 1).
m	$=$	$\frac{-4(1)^3 - 3(1)}{12(2)(1)^2 + 3(2) - 14}$	
m	$=$	$\frac{-7}{16}$	So the tangent line is $y = -\frac{7}{16}(x-2) + 1$

3. An astronaut on a foreign planet launches a ball from a height of 6 feet with an initial velocity of 14ft/s. The height of the ball can be measured using the function $h(t) = 0.5at^2 + 14t + 6$, where t is in seconds since the launch and a is the acceleration due to gravity of the foreign planet. Answer the following:

a. Determine the velocity function of the object. Your answer should be in terms of a and t.

The velocity is the derivative of the height function. $v(t) = h'(t) = at + 14$.

b. If the object reaches its maximum height at $t = 4$ seconds, determine the value for a. (What can you assume at the maximum height?)

The maximum height occurs when the velocity of the ball is zero (neither rising nor falling). Thus, plug in $t = 4$, $v = 0$, and solve for a. You should get $a = -14/4$ ft/s^2.

Notation convention dictates that vectors that either point up or right are positive. Gravity (which has a downward trajectory) is considered to be a negative value, so it makes sense that a is negative.

4. Air is being pumped into a spherical balloon at a rate of 11 cm^3/min. Determine the rate at which the radius is changing at the instant that the volume of the balloon is 36πcm^3.
You will need to use the volume of a sphere. You are given $\frac{dV}{dt}$ and you are computing $\frac{dr}{dt}$. After computing your derivative implicitly, you will need to solve for the radius.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \\ 11 &= 4\pi (3)^2 \frac{dr}{dt} \\ \frac{11}{4\pi (3)^2} &= \frac{dr}{dt} \end{aligned}$$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 36\pi &= \frac{4}{3}\pi r^3 \\ 27 &= r^3 \\ 3 &= r \end{aligned}$$

5. A cylindrical tank is being filled with water. The tank is 400ft high and its circular base has radius 70 ft. If the height is steadily increasing at a rate of 3 ft/min, determine the rate at which the volume is changing when the tank is halfway full.
Note that the radius of the tank does not change; it is constantly $r = 70$. This means that $\frac{dr}{dt} = 0$.

$$V = \pi r^2 h \qquad \text{Volume of cylinder.}$$

$$\frac{dV}{dt} = 2\pi r \left(\frac{dr}{dt}\right) h + \pi r^2 \left(\frac{dh}{dt}\right) \qquad \text{Remember to use the Product Rule.}$$

$$\frac{dV}{dt} = 2\pi(70)(0)(200) + \pi(70)^2(3) \qquad \text{Plug in your values .}$$

$$\frac{dV}{dt} = 14700\pi$$