Section 3.6: The Chain Rule

We start with a Pre-calculus review. Remember that the *composition* of two functions occur when a function is inside another function. Examples include: $\sqrt{5x-3}$, $(x-6)^3$, $\sin^2(x)$, etc.

Consider the following functions: $f(x) = x^3$, $g(x) = (2x+1)^3$, and $h(x) = \sin^3(x)$. Compute their derivatives.

General Power Rule:

$$\frac{d}{dx}\left[(u(x))^n\right] = n\left(u(x)^{n-1}\right) \cdot u'(x)$$

That is, if you take a function to a numeric exponent, then the derivative uses the power rule, as you have been doing, and then multiplies at the end by the derivative of the inner function.

Chain Rule:

$$\frac{d}{dx}\left[f(u(x))\right] = f'(u) \cdot u'(x) = \frac{df}{du} \cdot \frac{du}{dx}$$

It is imperative that you determine what your inside and outside functions are in your composition when computing derivatives using the Chain Rule.

Example: Compute the derivative of $j(x) = 5e^{2x^2+1}$.

Here, our inner function is $u(x) = 2x+1$, and our outer function is $f(u) = 5e^u$. To take the derivative of $j(x)$, you compute $f'(u)$ and $u'(x)$. By the constant multiple rule and the exponential rule, $f'(u) = 5e^u$. By the power rule, $u'(x) = 4x$. Thus,

$$j'(x) = f'(u) \cdot u'(x) = (5e^u) \cdot (4x) = 20e^u x = 20xe^{2x^2+1}$$

Notice that you're computing $j'(x)$, so you want your independent variable to be just x, not x and u, so you plug in.

Examples: Compute the derivative of the following functions.

$f(x) = \cos^7(x)$

$j(x) = 4x^7 \sec(5x^2)$

$g(x) = 3x^2 e^{-8x}$

$k(x) = \sqrt[4]{9x^2-7}$

$h(x) = \sin\left(\cos\left(3x^5-1\right)\right)$

$p(x) = \sec^2\left(\frac{2x+1}{3x-1}\right)$