Chapter 3 Overview: Tangent Lines, Derivatives, and Derivative Rules.

Section 3.1: Tangent Lines.
The **tangent line** of $f(x)$ at a point $x = a$ is the linear function: $y = m(x - a) + f(a)$. The value m is

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided that this limit exists. If $m = 0$, then we have what's called a horizontal tangent line.

Section 3.2: The Derivative As A Function.
The **derivative** of a function $f(x)$ with respect to the variable x is the function $f'(x)$ such that:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Note that when computing the tangent line at $x = a$, your slope $m = f'(a)$.

A function $f(x)$ is said to be **differentiable at a point** $x = a$ if $f'(a)$ exists. Similarly, a function $f(x)$ is said to be **differentiable** if $f'(x)$ exists for all values of x.

Derivative Notations.
There are many ways to denote derivatives.

Prime Notation: Suppose that $f(x)$ is a differentiable function. Then:
$f'(x)$ is considered its first derivative and $f''(x)$ is considered its second derivative.

Differential Notation: Suppose that $f(x)$ is a differentiable function. Then:
$\frac{df}{dx}$ is considered its first derivative and $\frac{d^2 f}{dx^2}$ is considered its second derivative.

Section 3.3/3.5: Derivative Rules and Properties.
As you can see, using the definition of derivative every time you are given a function can be quite troublesome, time consuming, and in some cases, algebraically impossible. This section introduces the shortcuts that you can take when computing derivatives.

We start with a list of derivative rules and properties. Let c be a real number, and let $u(x)$, $v(x)$, $n(x)$, $d(x)$, $u_1(x)$, $u_2(x)$, $u_3(x)$ all be differentiable functions.

Derivative Rule	Function	Derivative
Constant Function Rule:	$f(x) = c$	$f'(x) = 0$
Constant Multiple Rule:	$f(x) = c \cdot u(x)$	$f'(x) = c \cdot u'(x)$
Monomial Rule:	$f(x) = x^n$ (n a nonzero integer)	$f'(x) = n \cdot x^{n-1}$
Addition/Subtraction Rule:	$f(x) = u(x) \pm v(x)$	$f'(x) = u'(x) \pm v'(x)$
Power Rule:	$f(x) = x^n$ (n a nonzero real number)	$f'(x) = n \cdot x^{n-1}$.
Product Rule:	$f(x) = u(x) \cdot v(x)$	$f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$
Leibnitz Rule:	$f(x) = u_1(x)u_2(x)u_3(x)$ $f'(x) = u_1'(x)u_2(x)u_3(x) + u_1(x)u_2'(x)u_3(x) + u_1(x)u_2(x)u_3'(x)$	
Reciprocal Rule:	$f(x) = \frac{1}{d(x)}$	$f'(x) = \frac{d'(x)}{d(x) \cdot d(x)}$
Quotient Rule:	$f(x) = \frac{n(x)}{d(x)}$	$f'(x) = \frac{d(x) \cdot n'(x) - n(x) \cdot d'(x)}{d(x) \cdot d(x)}$.

Derivatives of More Functions.

Function	Derivative	Function	Derivative
$f(x) = e^x$	$f'(x) = e^x$		
$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$f(x) = \cos(x)$	$f'(x) = -\sin(x)$
$f(x) = \tan(x)$	$f'(x) = \sec^2(x)$	$f(x) = \cot(x)$	$f'(x) = -\cot^2(x)$
$f(x) = \sec(x)$	$f'(x) = \sec(x)\tan(x)$	$f(x) = \csc(x)$	$f'(x) = -\csc(x)\cot(x)$

Examples: Determine the first derivative of the following functions.

$$f(x) = 4\sin(x) + 5\cos(x) \qquad g(x) = x^3 e^x \sin(x)$$

$$f(t) = \frac{\cos(t) + t}{\sin(t) + 2t} \qquad g(w) = \frac{2\sin(w)\cos(w)}{2w + 1}$$

Examples: Simplify the following functions, then compute the first and second derivatives.

$$j(x) = \frac{\left(x^2 - 2\right)\left(x^3 + 5x\right)}{x} \qquad k(x) = e^x \tan(x)\csc(x)$$

Examples: Compute the derivative for each function below and determine its tangent line at $x = 1$.

$$f(x) = 4xe^x \qquad g(x) = \sqrt{x} + \sqrt[3]{x} + \frac{5}{3x}$$

$$f(t) = \frac{t^2 + 2t + 1}{t - 5} \qquad g(w) = \frac{w^8 + 3w}{5w + e^w}$$