**NAME:** ______________________________________________ **Group Work 4**

**Directions:** Remember that this is a no-calculator assignment. You may leave your answers unsimplified. Show all work if you want partial credit.

Compute $\frac{dy}{dx}$ for the following functions. **Please write in just your answer**.

1. $$y = \ln(x) + \cot(x) + \sec(x) + \arcsin(x)$$

$$\frac{dy}{dx} = \frac{1}{x} - \csc^2(x) + \sec(x)\tan(x) + \frac{1}{1-x^2}$$

2. $$y = \csc^3(18x^2 + 3x + 1) = \left[\csc(18x^2 + 3x + 1)\right]^3.$$

$$\frac{dy}{dx} = 3\left[\csc(18x^2 + 3x + 1)\right]^2 \cdot \left(-\csc(18x^2 + 3x + 1)\cot(18x^2 + 3x + 1)\right) \cdot 36x$$

3. $$y = \frac{e^x + 21x + 1}{x^2 + 7x}$$

$$\frac{dy}{dx} = \frac{(x^2 + 7x)(e^x + 21) - (e^x + 21x + 1)(2x + 7)}{(x^2 + 7x)^2}$$

4. $$y = \arctan(4x - 21)$$

$$\frac{dy}{dx} = \frac{1}{(4x + 21)^2 + 1} \cdot 4$$

5. $$y = \ln(\cos(37 - 11x))$$

$$\frac{dy}{dx} = \frac{1}{\cos(37 - 11x)} \cdot (-\sin(37 - 11x)) \cdot (-11)$$

6. $y = 27^{3x+5}$. Hint: Use a parentheses when you "bring the exponent down".

$$y = (3x + 5)\ln(27)$$

$$\frac{1}{y}\frac{dy}{dx} = 3\ln(27) + (3x + 5)(0) \qquad \longrightarrow \qquad \frac{dy}{dx} = y\,[3\ln(27)]$$

7. $$y = \frac{(2x + 9)^{11}\tan^4(x)}{\sin(x) + 3}$$

$$y = 11\ln(2x + 9) + 4\ln(\tan(x)) - \ln(\sin(x) + 3)$$

$$\frac{dy}{dx} = 11 \cdot \frac{1}{2x + 9} \cdot 2 + 4 \cdot \frac{1}{\tan(x)} \cdot \sec^2(x) - \frac{1}{\sin(x) + 3} \cdot \cos(x)$$

8. Use the equation below to determine the **first** and **second** derivative of $y(x)$. (Show your work.)

$$
\begin{aligned}
y^4 + 3x^4 &= 19 && \text{Use Implicit Differentiation term by term.} \\
4y^3\frac{dy}{dx} + 12x^3 &= 0 && \text{Solve for } \frac{dy}{dx}. \\
4y^3\frac{dy}{dx} &= -12x^3 && \text{Solve for } \frac{dy}{dx}. \\
\frac{dy}{dx} &= -\frac{12x^3}{4y^3} = -\frac{3x^3}{y^3} && \text{Simplify } \frac{dy}{dx}. \\
\frac{dy}{dx} &= -3x^3 \cdot y^{-3} && \text{Take derivative of } \frac{dy}{dx}. \\
\frac{d^2y}{dx^2} &= -3\left(3x^2\right)y^{-3} - 3x^3 \cdot \left(-3y^{-4}\frac{dy}{dx}\right) && \text{Simplify and substitute in } \frac{dy}{dx}. \\
\frac{d^2y}{dx^2} &= -9x^2y^{-3} + 9x^3y^{-4} \cdot \left(-3x^3 \cdot y^{-3}\right)
\end{aligned}
$$

9. If an arrow is shot upward on the moon with velocity of 33.2 m/s, its height (in meters) after t seconds is given by

$$h(t) = 33.2t - 0.83t^2.$$

(a) Determine the velocity of the arrow 9 seconds after launch.

To answer this question, you need to compute $v(9)$ where $v$ is the velocity function, which is the derivative of the position. Thus, $v(t) = 33.2 - 1.66t$. So $v(9) = 33.2 - 1.66(9) = 18.26$.

(b) Determine the impact velocity of the arrow.

Due to symmetries, if the ball is launched from the ground, the impact velocity will be the negative of the initial velocity. Thus, impact velocity is $-33.2$.

If you want to do this algebraically, set the position function equal to zero to determine impact time. You should get $t = 40$. Then plug this into the velocity function to get impact velocity: $v(40) = 33.2 - 1.66(40) = -33.2$.