NAME: ____ **Group Work 3**

Directions: Remember that this is a no-calculator assignment. You may leave your answers unsimplified. Show all work if you want partial credit.

Part 1: True/False. You will have parts of your exam as true/false questions, and they will be over the concepts of Chapter 2. For practice, here are the types of conceptual questions that you may see. Please write out the entire word in the answer box.

______________ 1. If $f(x)$ is defined at $x = a$, then $\lim_{x \to a} f(x)$ must also exist.
This is false. See the floor function at $x = 1$.

______________ 2. If $f(x)$ is continuous from the left at $x = a$, then $f(a)$ must exist.
This is true. The existence of $f(a)$ is required for continuity at $x = a$.

______________ 3. It is possible for a function to have infinitely many horizontal asymptotes.
This is false. A function can have at most two horizontal asymptotes.

______________ 4. It is possible for a function to have infinitely many vertical asymptotes.
This is true. Examples include $y = \sec(x)$, $y = \tan(x)$, etc.

______________ 5. It is possible for a rational function to have both a horizontal asymptote and a vertical asymptote.
This is true. See Quiz 3.

______________ 6. It is possible for a rational function to have both a horizontal asymptote and a slant asymptote.
This is false. For a rational function to have a slant asymptote, the degree of the numerator must be exactly one bigger than the degree of the denominator. And if the degree of the numerator is bigger than the degree of the denominator, there is no horizontal asymptote.

______________ 7. It is possible for a rational function to have both a slant asymptote and a vertical asymptote.
This is true. See $j(x)$ from Section 2.6.

______________ 8. If $f(x)$ is a polynomial, then it must be continuous at $x = 4$.
This is true. If $f(x)$ is a polynomial, it is continuous everywhere, which means it is continuous at $x = 4$.

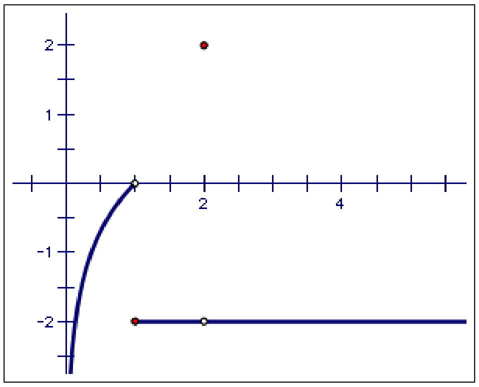
______________ 9. If $f(x)$ and $g(x)$ are continuous everywhere, then $(f + g)(x)$ must also be continuous everywhere.
This is true. It is a property of continuous functions.

Part 2: Examples You might also be asked to show your conceptual understanding by supplying an example. Please provide a function (not a graph) that satisfies the following criteria:

10. A function $f(x)$ that has an infinite discontinuity at $x = 0$, a jump discontinuity at $x = 1$, and a removable discontinuity at $x = 2$. [Hint: Think piecewise.]

Note: Answers will vary.

$$f(x) = \begin{cases} \ln(x) & \text{for } 0 < x \leq 1 \\ -2 & \text{for } 1 < x < 2 \\ 2 & \text{for } x = 2 \\ -2 & \text{for } x > 2 \end{cases}$$



11. Use the Intermediate Value Theorem to determine an interval $[0, b]$ such that the equation $5 = x + \cos(\pi x)$ has a solution in your interval.

Let $f(x) = -5 + x + \cos(\pi x)$ and $c = 0$. Note that $f(x)$ is a polynomial $(-5 + x)$ minus a trig function (with a period alteration) that is continuous everywhere. Thus $f(x)$ is continuous everywhere and the IVT will apply for any interval.

Note that $f(0) = -4$, so you want a b value such that $f(b)$ is positive, thus c will be in between $f(b)$ and $f(a)$. Through trial and error, you should get $b = 6$: $f(6) = -5 + 6 + cos(6\pi) = 2$. Thus, the IVT guarantees that $f(x)$ will equal 0 on the interval $[0, 6]$.

Some of you stumbled upon the answer, which is $x = 4$, which makes the IVT useless.

12. Let $f(x) = x^3 + x - 1$. Use the definition of derivative to compute $f'(x)$. Then determine the tangent line of $f(x)$ at the point $x = 2$.

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} && \text{Apply Definition of Derivative.} \\
&= \lim_{h\to 0} \frac{(x+h)^3 + (x+h) - 1 - (x^3 + x - 1)}{h} && \text{Clean up numerator.} \\
&= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^2 + x + h - 1 - x^3 - x + 1}{h} && \text{Cancel terms.} \\
&= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^2 + h}{h} && \text{Divide each numerator term by } h. \\
&= \lim_{h\to 0} 3x^2 + 3xh + h + 1 \\
&= 3x^2 + 1
\end{aligned}
$$

Now to compute the tangent line, you will need to use the point-slope formula. $x_1 = a = 2$. $y_1 = f(a) = (2)^3 + 2 - 1 = 9$. $m = f'(a) = 3(2)^2 + 1 = 13$. Thus, your tangent line is:

$$y = 13(x - 2) + 9$$