NAME: ____ **Group Work 2**

Directions: Remember that this is a no-calculator assignment. You may leave your answers unsimplified. You may work on this on your own or with a group, but you need to submit your own work. Show all your work if you want partial credit.

1. Compute the following limits algebraically. Show your work.

(a) $\lim_{t\to 0} \frac{\sin^2(4t)}{8\tan(13t^2)} = \lim_{t\to 0} \frac{\sin(4t)\sin(4t)}{8} \frac{\cot(13t^2)}{1} = \lim_{t\to 0} \frac{\sin(4t)\sin(4t)\cos(13t^2)}{8\sin(13t^2)}$

$= \lim_{t\to 0} \frac{4t}{4t}\frac{4t}{4t}\frac{\sin(4t)\sin(4t)\cos(13t^2)}{8\sin(13t^2)}\frac{13t^2}{13t^2} = \lim_{t\to 0} \frac{16}{13}\frac{t^2}{t^2}\frac{\sin(4t)}{4t}\frac{\sin(4t)}{4t}\frac{\cos(13t^2)}{8}\frac{\sin(13t^2)}{13t^2}$

$= \frac{13}{16} \cdot 1 \cdot 1 \cdot 1 \cdot \frac{1}{8} \cdot 1 = \frac{16}{104}$

(b) $\lim_{x\to -\infty} \frac{5+e^x}{e^x - 7}$. Hint: Analyze the graph of e^x and its behavior as $x \to -\infty$.

As $x \to -\infty$, e^x goes to zero. Thus the numerator will limit to 5 and the denominator will limit to -7.

(c) $\lim_{x\to\infty} \frac{5}{e^{2x}} = 0$

As $x \to \infty$, $e^{2x} \to \infty$.
Thus the denominator gets infinitely large while the numerator stays a 5.
Therefore, this limit is zero.

(d) $\lim_{x\to\infty} \frac{5+7e^{2x}}{4e^{2x}+9} = \lim_{x\to\infty} \frac{\frac{5}{e^{2x}} + \frac{7e^{2x}}{e^{2x}}}{\frac{4e^{2x}}{e^{2x}} + \frac{9}{e^{2x}}} = \lim_{x\to\infty} \frac{\frac{5}{e^{2x}} + 7}{4 + \frac{9}{e^{2x}}}$

And from part (c), we know that the red pieces will go to 0 while the rest of it stays constant. Thus, the limit is $\frac{7}{4}$.

2. Provide an explicit function (not a graph) that satisfies the following criteria. No justification is required.

(a) A rational function $f(x)$ such that it has a horizontal asymptote at $y = \frac{15}{37}$ and a vertical asymptote at $x = 95$.

Answers will vary, but most people went with $f(x) = \frac{15x}{37(x-95)}$

(b) A non-linear function $g(x)$ that has a slant asymptote at $y = 2x+3$ and a vertical asymptote at $x = 4$.

Due to the vertical asymptote requirement, the denominator has to be at least $x-4$ or any polynomial multiple. For the sake of simplicity, use $x-4$. You then have to find a numerator such that long division will result in the desired slant asymptote. Anything in the form of $2x^2+5x+c$ works.

(c) A rational function $h(x)$ that has no horizontal, slant, or vertical asymptotes.

Any polynomial works. Remember that all polynomials are rational.

3. Use the conjugate trick to compute the following limit. Show all your work.

$$\lim_{h\to 0}\frac{\sqrt{7x+7h+5}-\sqrt{7x+5}}{13h} = \lim_{h\to 0}\frac{\sqrt{7x+7h+5}-\sqrt{7x+5}}{13h}\left(\frac{\sqrt{7x+7h+5}+\sqrt{7x+5}}{\sqrt{7x+7h+5}+\sqrt{7x+5}}\right)$$

$$= \lim_{h\to 0}\frac{7x+7h+5-(7x+5)}{13h(\sqrt{7x+7h+5}+\sqrt{7x+5})}$$

$$= \lim_{h\to 0}\frac{7x+7h+5-7x-5}{13h(\sqrt{7x+7h+5}+\sqrt{7x+5})}$$

$$= \lim_{h\to 0}\frac{7h}{13h(\sqrt{7x+7h+5}+\sqrt{7x+5})}$$

$$= \lim_{h\to 0}\frac{7}{13(\sqrt{7x+7h+5}+\sqrt{7x+5})}$$

$$= \frac{7}{13(\sqrt{7x+5}+\sqrt{7x+5})}$$