Section 5.5: The Indefinite Integral and the Substitution Rule.
We start with a definition. The set of all antiderivatives of a function f is the **the indefinite integral of f with respect to x**, denoted by:

$$\int f(x)dx$$

NOTE: It is important that you do not leave out the dx. Later if/when you go further into Calculus, this component will become more significant.

For example, the indefinite integral of $f(x) = 2x$ is $x^2 + c$, or $\displaystyle\int 2x dx = x^2 + c$.

We then proceed with an important theorem:

> **Theorem 5 - The Substitution Rule**: If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then
>
> $$\int f(g(x))g'(x)dx = \int f(u)du.$$

This is essentially the reverse chain rule. When you were taking derivatives, any time a function is composed into another, you had to apply the chain rule. We now have to work backwards: You must come up with the compositions, check to see if it works, and then proceed.

When doing u-substitution, there are a few tips to consider:

- Simplify as much as possible! You could get away with this with the derivative rules, but now it is a complete must for integrals. (This applies to all integration in general.)
- The first step is to recognize your "inner" function. This is usually apparent: It's usually under a square root, inside a trig function, the exponent of an exponential function, etc.
- You need to completely transform your integral from an integral in terms of x to an integral in terms of u. If there is a mix of x and u terms, you are doing something wrong. Check your work. You may have to try a new "inner function". [NOTE: Keep in mind that u-substitution might not work.]
- When doing u-substitution, you will be simultaneously working with integrals and derivatives. Be careful not to mix them up.
- Your du term should *always* be in the numerator. A du term in the denominator means you either made an algebraic mistake, or you need to pick a new u.
- After you finish applying the antiderivative rules, remember to do two things at the end: You have to back substitute and you need your $+\ c$.

Examples: Compute the following indefinite integrals.

1. $\displaystyle\int \left((4x+9)^{13}\right) dx$
2. $\displaystyle\int \left(3x(x^2+2)^5\right) dx$
3. $\displaystyle\int \left(\frac{9x^2}{(x^3+10)^4}\right) dx$
4. $\displaystyle\int \left(5x\sqrt{x^2+11}\right) dx$
5. $\displaystyle\int \left(\frac{(\ln(6x))^4}{5x}\right) dx$
6. $\displaystyle\int \left(5x^3 e^{10x^4}\right) dx$
7. $\displaystyle\int \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx$
8. $\displaystyle\int x^{1/3}\sin(x^{4/3}-8)dx$
9. $\displaystyle\int \left(6x^2\sec^2(9x^3+4)\right) dx$
10. $\displaystyle\int \cot(x)dx$