Section 5.5: Trig Functions Translations and Dilations.

As like the last unit, I will use the notation trig(θ) to mean all six trig functions. This worksheet deals with dealing with functions of the form

$$f(x) = a \text{ trig}(bx + c) + d$$

where a, b, c, and d are real numbers, with $a,\ b \neq 0$.

Vertical Shift: $f(x) = \text{trig}(x) + d$
This should be a familiar graph movement: you will move your graph up or down depending on the value of d.

Vertical Stretch: $f(x) = a \text{ trig}(x)$
This again should be familiar. When you multiply a function by a real number coefficient, you will either dilate or constrict your graph vertically. The following chart explains how vertical stretches affect each trig function:

Function	Range	Notes
$a\sin(x)$ and $a\cos(x)$	$[-a, a]$	The value $\|a\|$ is the **amplitude**
$a\tan(x)$ and $a\cot(x)$	$(-\infty, \infty)$	The graph either flattens (if $\|a\| < 1$) or steepens (if $\|a\| > 1$.)
$a\sec(x)$ and $a\csc(x)$	$(-\infty, -a] \cup [a, \infty)$	The missing gap gets wider.

The **amplitude** of a function is a measure of variability of the range of your function. Since $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ have ranges that go to $-\infty$ and $+\infty$, these four functions do not have an amplitude.

For a function $f(x) = a\sin(x)$ or $f(x) = a\cos(x)$, the amplitude will be $|a|$. Note that the amplitude is always positive. What does it mean when a is negative?

Horizontal Stretch: $f(x) = \text{ trig}(bx)$
A horizontal movement occurs when you alter the "inside part" of your function. The horizontal stretch will affect the period of a function. The **period** is a measure (in the x-direction) of how long it takes for the graph to repeat itself. The following chart explains how horizontal stretches affect each trig function:

Trig Function:	$\sin(bx)$ and $\cos(bx)$	$\tan(bx)$ and $\cot(bx)$	$\sec(bx)$ and $\csc(bx)$
Period			

Horizontal Shifts: $f(x) = \text{trig}(bx + c)$
A horizontal shift (also called **phase shift**) occurs when you further alter the "inside part" of your function. Phase shifts, like amplitude, are generally only talked about when dealing with $\sin(x)$ and $\cos(x)$.
The phase shift formula for both $\sin(bx + c)$ and $\cos(bx + c)$ is $-\dfrac{c}{b}$

Examples:

1. Compute the amplitude, period, and phase shifts of the following functions. Then determine the range.
   $f(x) = \sin\left(x + \dfrac{5\pi}{9}\right)$ $\quad$ $g(x) = 7\cos(8x - \pi) - 3$ $\quad$ $h(x) = -\dfrac{1}{6}\cos\left(\dfrac{4}{5}x\right) + 3$
2. Determine the range of the following functions.
   $y = -5|\sin(7x)| + 6$ $\quad$ $y = 4\sec(3x) + 2$ $\quad$ $y = 3 - \cos^2(8x)$
3. Let $f(x) = 3\sin(13x)$. Determine the coordinates of the first maximum turning point of the graph of $f(x)$ in the interval $(0, 2\pi)$.
4. Let $g(x) = -5\cos\left(\dfrac{x}{6}\right)$. Determine the coordinates of the first maximum turning point of the graph of $g(x)$ in the interval $(0, 2\pi)$.

Obtaining Amplitude, Period, Phase Shift, and Vertical Shift from a Graph:
The previous side deals with obtaining Amplitude, Period, Phase Shift, and Vertical shift from a function. We now analyze it graphically.

To compute the amplitude and vertical shifts of a graph, you have the following formula:

$$\text{Amplitude} = \frac{\text{highest } y\text{-value} - \text{lowest } y\text{-value}}{2} \qquad \text{Vertical Shift} = \frac{\text{highest } y\text{-value} + \text{lowest } y\text{-value}}{2}$$

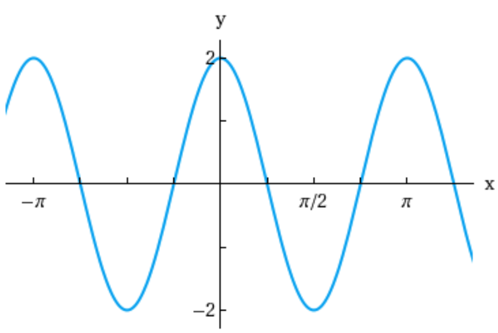
Notice that the two formulas are very similar. Make sure you know which is which.

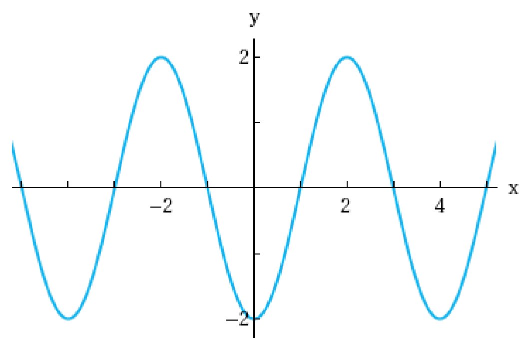
There is no set formula for period. You are just determining how long (in the x-direction) it takes for the graph to repeat. The easiest way to do this is by analyzing the distance between consecutive high points (for sine and cosine), distance between horizontal asymptotes (for tangent and cotangent), or how long it takes for a graph to complete a rightside up/upside down parabolic graph (for secant and cosecant).

For our class, the phase shift will be the *first increasing x-intercept left of the y-axis.*

Examples:

1. Let $f(x) = a\sin(x) + d$, with a positive. Determine the values of a and d that will make $f(x)$ oscillate between -3 and 13.
2. The graph of a sine function with a positive coefficient is shown below, left. Determine the amplitude, period, and phase shift. (The phase shift is the first negative zero that occurs before a maximum.) Then write the equation in the form $y = a\sin(bx + c)$ for $a > 0$, $b > 0$, and the least positive real number c.





3. The graph of a sine function with a positive coefficient is shown above, right. Determine the amplitude, period, and phase shift. (The phase shift is the first negative zero that occurs before a maximum.) Then write the equation in the form $y = a\sin(bx + c)$ for $a > 0$, $b > 0$, and the least positive real number c.
4. A tidal wave has total height 50 ft. Every 30 minutes, a new wave crest appears, so the period is 30 minutes. We measure the distance y from the sea level to the top of the wave and model this distance y with a function $y = a\cos(bt)$, with $a > 0$, $b > 0$, and t is in minutes since the last crest and the distance y is in feet. Determine a and b.
5. Suppose that $f(x) = a + \sin(bx)$ with a and b are constants, and $-5.5 < b < 5.5$. Determine a and b if:

   (a.) $f(0) = 5$ and $f\left(\frac{\pi}{11}\right) = 5.5$ (b.) $f(0) = 3$ and $f\left(\frac{\pi}{11}\right) = 3 - \frac{\sqrt{2}}{2}$
6. Suppose that $g(x) = a + \tan(bx)$ where a and b are constants, and $-3.5 < b < 3.5$. Determine a and b if:

   (a.) $g(0) = 1$ and $g\left(\frac{\pi}{7}\right) = \frac{\sqrt{3}}{3} + 1$ (b.) $g(0) = 2$ and $g\left(\frac{\pi}{7}\right) = 1$
7. Suppose that $h(x) = a + \cos(bx)$ where a and b are constants, and $0 < b < 6$. Determine a and b if:

   (a.) $h(0) = 3$ and $h\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{2}} + 2$ (b.) $h(0) = 4$ and $h\left(\frac{\pi}{6}\right) = 3 - \frac{\sqrt{3}}{2}$