Section 4.6B: Optimization.

Set-up Functions: For the following word problems, determine the optimized function (in terms of one variable) as well as its relevant domain.

1. Three large squares of tin, each with edges 100 cm long, have four small equal squares (of at least 1cm in length) cut from their corners. All twelve resulting small squares have the same size. The three large cross-shaped pieces are then folded and welded to make boxes (of at least 1 cm in length) with no tops, and the twelve small squares are used to make two small cubes. How should this be done to maximize the total volume of all five boxes?

2. A Norman Window is constructed with a rectangular glass pane surmounted by a semicircular glass pane. Determine the largest possible area for such a window if the external perimeter must be 200 cm.

Optimize Completely:

3. What are the dimensions of the lightest open-top right circular cylinder can that will hold a volume of 500π cubic cm.? (HINT: You are minimizing Surface Area.)

4. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 in. What dimensions will give a box with a square end the largest possible volume?

5. A rectangular storage container with an open top is to have a volume of 18 cubic meters. The length of its base is twice the width x. Material for the base costs 13 dollars per square meter. Material for the sides costs 5 dollars per square meter. Answer the following:

   (a) Determine the cost function $C(x)$.

   (b) Find the cost for the cheapest such container. Round your answer to the nearest cent.

6. A fence is to be built to enclose a rectangular area of 280 square feet. The fence along three sides is to be made of material that costs 5 dollars per foot, and the material for the fourth side costs 12 dollars per foot. Find the dimensions of the enclosure that is most economical to construct.