Section 4.6: Optimization.

Question: What is the smallest perimeter possible for a rectangle whose area is 49 in^2?

Your first task is to create a function in terms of one variable, then finding its relevant domain. For this problem, you have a rectangle. Let l = length and w = width.

$P = 2l + 2w$. Using the area constraint $49 = l \cdot w$, you get $\frac{49}{w} = l$, and thus $P(w) = 2\left(\frac{49}{w}\right) + 2w$.

The next step is to determine the relevant domain. There is one implied constraint: w must be positive, otherwise you have no area. In this case, the domain is $(0, \infty)$.

Now you're looking for critical points. Take the derivative and solve for when the derivative is undefined ($w = 0$) and when the derivative is zero ($w = \pm 7$). Only one is relevant: $w = 7$.

Now you want to prove that you have a global min. In a closed interval, you would evaluate CPs and EPs. In an open interval, you would use the first derivative test: make your sign graph.

For the interval $(0, 7)$, $P'(1) = 2 - \frac{98}{1^2} = -96$. Thus, $P(w)$ is decreasing on this interval. Similarly, on the interval $(7, \infty)$, $P'(10) = 2 - \frac{98}{10^2} = 1.02$. Thus, $P(w)$ is increasing on this interval.

Examples:

1. A rectangle has its base on the x axis and its upper two vertices on the parabola $y = 18 - x^2$. What is the largest area the rectangle can have?
2. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand fence. With 200 m of wire at your disposal, what is the largest are that you can enclose?
3. You are designing a rectangular poster to contain 80 sq in of printing with a 4 in margin at the top and bottom and 2 in margin at each side. What overall dimensions will minimize the amount of paper used?
4. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius $r = 7$?
5. What are the dimensions of the lightest open top right circular cylinder that will hold a volume of 500π cm^3? Hint: you are minimizing surface area.
6. Jane is $c = 4$ miles offshore in a boat and wishes to reach a coastal village $b = 7$ miles down a straight shoreline from the point nearest the boat. She can row 2 mph and walk 5 mph. Where should she land her boat to reach the village in the least amount of time?