Section 4.2: The Mean Value Theorem Revisited.
Mean Value Theorem:
Suppose that $y = f(x)$ is continuous on the *closed interval* $[a, b]$ and differentiable *in its interior* (a, b). Then there is at least one point c in (a, b) at which:

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Basically, this tells us that the **average rate of change** of $f(x)$ on an interval will be the same as the **instantaneous rate of change** of $f(x)$ at some point inside the interval, provided that the Mean Value Theorem applies.

Corollary 1:
If $f'(x) = 0$ at each point x of an open interval (a, b), then $f'(x) = C$ for all x in (a, b) where C is a constant.

Corollary #1 tells us what we are hoping: *If a function has a derivative of zero, then the original function must have been some constant.*

Corollary 2:
If $f'(x) = g'(x)$ at each point x in an open interval (a, b) then there exists a constant C such that $f(x) = g(x) + C$ for all x in (a, b).

Corollary #2 tells us that if two functions have the same derivative, then they will differ by only a constant. Thus, *there are infinitely many functions that have the same derivative, and that this infinite set of functions will only differ by a constant.* For example, there are many functions whose derivative is $2x$. (Examples of such functions are x^2, $x^2 + 8$, $x^2 - 7$, etc.)

Section 4.8: Antiderivatives.
Given a function $f(x)$ that is differentiable on an interval I, then a function $g(x)$ is called the **antiderivative** of $f(x)$ if $g'(x) = f(x)$.

As mentioned in Section 4.2, given a function $f(x)$, there are infinitely many antiderivatives of $f(x)$, but they all differ from each other by a constant.

Anti-Derivative Power Rules: For a real number n, we have:

- If $f'(x) = n$, then $f(x) = nx + C$.
- If $f'(x) = 0$, then $f(x) = C$.
- If $f'(x) = x^n$, then $f(x) = \dfrac{x^{n+1}}{n+1} + C$ (as long as n $\neq -1$.)

Anti-Derivative Exponential/Logarithmic Rules:

- If $f'(x) = e^x$, then $f(x) = e^x + C$
- If $f'(x) = \dfrac{1}{x}$, then $f(x) = \ln|x| + C$.

Anti-Derivative Trig Rules:

- If $f'(x) = \cos(x)$, then $f(x) = \sin(x) + C$
- If $f'(x) = \sin(x)$, then $f(x) = -\cos(x) + C$
- If $f'(x) = \sec^2(x)$, then $f(x) = \tan(x) + C$
- If $f'(x) = \csc^2(x)$, then $f(x) = -\cot(x) + C$
- If $f'(x) = \sec(x)\tan(x)$, then $f(x) = \sec(x) + C$
- If $f'(x) = \csc(x)\cot(x)$, then $f(x) = -\csc(x) + C$

Examples: Compute an anti-derivative of the following functions:

$f'(x) = x^2 + 3x - 4 + \dfrac{4}{x^2}$　　$g'(x) = \sin(x) + \csc^2(x)$　　$h'(x) = 3e^x + \dfrac{5}{3x}$

$f'(x) = (3x - 1)(2x + 3)$　　$g'(x) = x\sqrt{x} - \dfrac{13}{5\sqrt[3]{x}}$　　$h'(x) = \dfrac{2x^4 - 3x^3 + 5}{7x^2}$

Section 4.8B: Initial Value Problems Involving Anti-Derivatives.
When given the derivative $f'(x)$, remember that it is impossible to determine what the initial function $f(x)$ is. We can determine an antiderivative of $f'(x)$, and we know that this antiderivative and $f(x)$ will differ by just a constant.

However, when we are given extra information about $f(x)$, we can pinpoint exactly which antiderivative is $f(x)$.

Examples: Compute the original functions for each given information.

1. $\dfrac{df}{dx} = 3x^3 + \dfrac{2}{x^2}$; $f(1) = 1$
2. $\dfrac{dg}{dx} = 6e^x$; $g(0) = 10$
3. $\dfrac{dh}{dx} = 8\cos(x)$; $h(0) = 3$

Kinematics Revisited: We now go back and review the Kinematics from Section 3.1, where the topics of acceleration, velocity, and position were discussed. Remember that:

Position: $s(t)$ **Velocity**: $v(t) = s'(t)$ **Acceleration**: $a(t) = v'(t)$ or $s''(t)$

So by working backwards, *velocity is the antiderivative of acceleration*, and *position is the antiderivative of velocity*.

Given a constant acceleration $a(t) = a$, we have:

Acceleration:	$a(t)$	$=$	a	
Velocity:	$v(t)$	$=$	Anti-Derivative of Acceleration.	
		$=$	$at + v_0$	[v_0 is called the initial velocity]
Position:	$s(t)$	$=$	Anti-Derivative of Velocity.	
		$=$	$0.5at^2 + v_0t + s_0$	[s_0 is called the initial position]

Side Note: If your acceleration is not constant, then you will have to do the integrations as needed. This might show up on WebWork and on the book, and you will probably encounter these in Calculus II, but for our class, in terms of quizzes and the exam, *we will be working with a constant acceleration with kinematic problems.*

Side Note II: For those of you that know/use those kinematic formulas from Physics, note that those are for **METRIC** values. I will almost always give you imperial (English) units. You can either use your formulas and convert, or go with our notes.

Word Problems: Unless otherwise stated, use -32 ft/sec^2 as acceleration due to gravity.

1. When Alex shot a marble straight upward from ground level with his slingshot, it reached a maximum height of 400 ft. What was the marble's initial velocity?
2. A ball is dropped from near the top of the Empire State Building, at a height 960 ft above 34th Street. How long does it take for the ball to reach the street and with what velocity does it strike the street?
3. Kosmo throws a baseball straight downward from the top of a tall building. The initial speed of the ball is 25 ft/s. It hits the ground with a speed of 153 ft/s. How tall is the building?
4. The acceleration due to gravity on Venus is roughly 30 feet per second. An astronaut is standing on top of his 140 foot high space rocket. He throws the ball up in the air and it lands on the ground 4 seconds later. At what velocity did he throw the ball?
5. A car's breaks are applied when the car is moving at 60 mi/hr (88 ft/s). The brakes provide a constant deceleration of 44 ft/s^2. how far does the car travel before coming to a stop?
6. A car traveling at 60 mi/hr (88 ft/s) skids for 132 ft after its brakes are applied. The deceleration provided by the braking system is constant. What is this value?