Section 3.8, 3.9: Derivatives of Inverse Functions.

Suppose that $f(x)$ and $g(x)$ are inverses with $f(x)$ differentiable. That means that $f(g(x)) = x$.

Using implicit differentiation and solving for $\frac{dg}{dx}$, you get:

$$\frac{dg}{dx} = \frac{1}{f'(g(x))}$$

We then apply this to compute the derivative of seven more functions: $\ln(x)$ as well as the arctrig functions.

Examples: Use the Chain rule, as well as the derivative of $ln(x)$ to compute the derivatives of following functions:

(a) $f(t) = \ln(4t^2)$

(b) $g(\theta) = \ln(\sin(\theta))$

Examples: Use properties of logarithms to simplify the following equations. Then use implicit differentiation to compute dy/dx.

(a) $y = \ln\left(\frac{7x - 15}{x\sqrt[7]{x^2+1}}\right)$

(b) $y = (-9x^2 + 4)^6(3x^2 + 7)^4$

(c) $y = \left(\sqrt[3]{t}\right)^t$

(d) $x^y = y^x$

Inverse Trig Derivatives: The proof for the derivative of $\arcsin(x)$ will be shown in class. The three arctrig derivatives you are expected to know are:

$f(x) = \arcsin(u)$	$\frac{df}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$
$f(x) = \arccos(u)$	$\frac{df}{dx} = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$
$f(x) = \arctan(u)$	$\frac{df}{dx} = \frac{1}{u^2+1} \cdot \frac{du}{dx}$

Examples: Compute the derivative of the following functions:

(a) $y = \arcsin(5x^2)$

(b) $y = \arccos\left(\frac{6}{x^2}\right)$

(c) $y = \ln(\arctan(8x))$

(d) $y = \arctan(\ln(8x))$