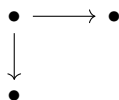


- Let I denote the category



and let \mathcal{A} be an abelian category. Prove that the colimit of any functor $F : I \rightarrow \mathcal{A}$ exists. (This colimit is called a *pushout*.)

- (Do not hand in) Weibel, Exercise 3.2.1
- Weibel, Exercise 3.2.2, 3.4.1
- Let A and B denote cochain complexes of objects in an abelian category \mathcal{A} . Let $\mathcal{H}om(A, B)$ denote the total Hom cochain complex. Prove $\text{Hom}_{\mathbf{K}(\mathcal{A})}(A, B[n]) = H^n(\mathcal{H}om(A, B))$.
- Consider the category \mathbf{Ab} of abelian groups. Compute $\text{Hom}_{\mathbf{T}}(\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}/2\mathbb{Z}[n])$ for all n for $\mathbf{T} = \mathbf{K}(\mathbf{Ab})$ and $\mathbf{T} = \mathbf{D}(\mathbf{Ab})$.
- (Not to hand in) Let $R = \mathbb{C}[\mathfrak{t}, \mathfrak{t}^{-1}]$, the ring of Laurent polynomials in one variable. An R -module is the same as a complex vector space V equipped with an automorphism $\mathfrak{t} : V \rightarrow V$. Given an R -module M , let $M^{\mathfrak{t}}$ be the space of \mathfrak{t} -invariants in M :

$$M^{\mathfrak{t}} = \{m \in M \mid \mathfrak{t}m = m\}.$$

Given two R -modules M, N , consider the space $\text{Hom}_{\mathbb{C}}(M, N)$ of linear transformations between them. This can be made into an R -module as follows: for $f \in \text{Hom}_{\mathbb{C}}(M, N)$, let $(\mathfrak{t} \cdot f)(m) = \mathfrak{t}f(\mathfrak{t}^{-1}m)$. Show that there is a natural isomorphism

$$\text{Hom}_R(M, N) \cong \text{Hom}_{\mathbb{C}}(M, N)^{\mathfrak{t}}.$$

- Now prove the derived version of the previous result.
 - Explain how to define $R\text{Hom}_{\mathbb{C}} : \mathbf{D}^-(R\text{-mod})^{op} \times \mathbf{D}^+(R\text{-mod}) \rightarrow \mathbf{D}^+(R\text{-mod})$.
 - Let $J : R\text{-mod} \rightarrow \mathbb{C}\text{-mod}$ be the functor $J(M) = M^{\mathfrak{t}}$. Show that J is left exact.
 - Prove that for $M \in \mathbf{D}^-(R\text{-mod})$ and $N \in \mathbf{D}^+(R\text{-mod})$, there is a natural isomorphism

$$R\text{Hom}_R(M, N) \cong RJ(R\text{Hom}_{\mathbb{C}}(M, N)).$$

Hint: You will need to show that if $A, I \in R\text{-mod}$ with I injective, then $\text{Hom}_{\mathbb{C}}(A, I)$ is also an injective R -module.