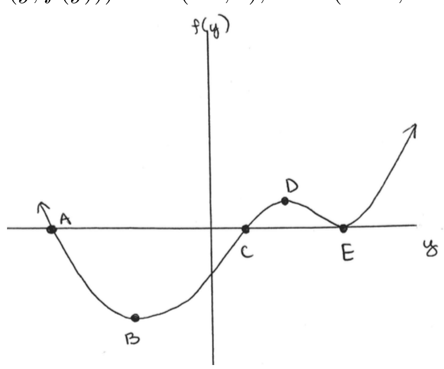
**Justify your answers and show work** to receive full credit. A correct answer with no work or explanation will not receive full credit.

1. **True / False** If a solution to the differential equation $\frac{dy}{dt} = f(y)$ (with $f$ continuously differentiable) is increasing at time $t = t_0$, then that solution must be increasing for all $t$.
2. **True / False** The lines $t = 0$ and $y = 5$ are equilibrium solutions for the differential equation $\frac{dy}{dt} = t^2(y-5)$.
3. **True / False** The solution to the initial value problem $\frac{dy}{dt} = (y^2 - 16)(5\sin t + 4), y(0) = -3$ satisfies $y(t) > -4$ for all $t$.
4. Solve the initial value problem $\frac{dy}{dt} = t^2(3y-1), y(0) = 5$.
5. Find the general solution the the differential equation $\frac{dy}{dt} = y^2$. (*Hint*: does your solution work for any possible initial value?)
6. Use Euler's method to approximate $y(0.3)$ for the differential equation $\frac{dy}{dt} = 4y^3 + 20t$ with initial value $y(0) = -0.5$ and step size $\Delta t = 0.1$. (Include a table with $t$ and $y$ values for each iteration of Euler. You may round all numbers to two decimal places.)
7. An up-and-coming clothing design company is studying the demographics of its employee population. They've noticed that each year (on average), 20 new people are hired, 5 of which speak a second language fluently. On the other hand, they've noted that 10 employees quit each year. Assume the company employs 20 people initially, and only two of them speak a second language.
   (a) How many employees does the company have after $t$ years? When does the company have exactly 100 employees?
   (b)Let $S$ denote the number of employees that speak a second language. What is the initial value problem modeling the population $S$?
   (c) What percentage of employees speak a second language when the company has exactly 100 employees?
8. Consider the graph of (continuously differentiable) $f(y)$ shown below with marked points (in the form $(y, f(y))$) $A = (-5, 0)$, $B = (-2.8, -8)$, $C = (1, 0)$, $D = (3.3, 2.6)$, and $E = (7, 0)$.




   (a) Sketch the phase line for $\frac{dy}{dt} = f(y)$. Identify any sinks, sources, and nodes.
   (b) Sketch solutions with initial values $y(0) = -6, y(0) = 0, y(0) = -5$. Clearly mark the initial value on the corresponding solution. (Note that your graph is only approximate since your interpretation of $f$ is approximate. However, the initial value and the information you convey about long term behavior of solutions is *not* approximate.)
   (c) Let $\alpha$ be a parameter, and consider the one-parameter family of differential equations $\frac{dy}{dt} = f(y) + \alpha$. Do you expect bifurcation values for $\alpha$? If so, what are they?