

Basic Definitions

You should be able to give precise descriptions of all of the following. You should also have in mind *examples* that satisfy the definition and some that do not. Throughout, let F denote a field.

- All words/concepts from Exam I.
- (for general rings) homomorphism, isomorphism, surjective, injective, image, ideal
- Be able to work with the ring $F[x]$. What are its elements? What are the operations?
- All words associated to $F[x]$. i.e. degree, monic, gcd, irreducible, root
- greatest common divisor of two polynomials
- Distinction between $f(x) \in F[x]$ and a polynomial function $f : F \rightarrow F$.
- congruence modulo $p(x)$, congruence classes
- $F[x]/(p(x))$. You should be able to describe $F[x]/(p(x))$ as a set (what are its elements?) and as a ring (what are the operations?). (See Cor. 5.5)
- field extension

Big Theorems/Ideas

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class, describe components and main ideas of the proof.

- Be able to verify a map is or isn't a homomorphism/isomorphism.
- $F[x]$ is an integral domain
- degree properties
- What are units of $F[x]$? Zero divisors? Be able to justify.
- Division algorithm in $F[x]$
- Theorem 4.14: Unique factorization of polynomials in $F[x]$.
- Cor. 4.18: Irreducible with degree bigger than 1 implies no roots; What about the converse?
- Units and zero divisors in $F[x]/(p(x))$; Thm. 5.9, 5.10.
- Completely understand example $\mathbb{R}[x]/(x^2 + 1)$.
- Be able to verify something is or isn't an ideal; Theorem 6.1

Note that...

This is a proofs-based class. Therefore, there will be questions apart from computations and stating definitions requiring you to assemble ideas in fresh ways to establish statements you have not already seen before. The proof problems on your homework assignments are representative of the sorts of proofs you might be asked on an exam.

Extra problems

You should carefully review all solutions to assigned homework. In addition, I recommend looking at the following problems from your textbook:

Section 3.3: #15

Section 4.2: #4; Section 4.3: #10, 12; Section 4.4: #28

Section 5.1: #3; Section 5.2: #5, 9, 14; Section 5.3: #2, 3

Section 6.1: # 1, 2, 9