

Instructor: Dr. Laura Rider
Office: 633 Boyd
Office Hours: 1:15-2:15M, 2-3Th (tentative), and by appointment
Email: laurajoy@uga.edu
Course Webpage: <https://faculty.franklin.uga.edu/laurajoy/content/geomrep>
Meetings: 12:20 - 1:10 MWF in 410 Boyd

Course Description For the purposes of this course, “Geometric Representation Theory” means using singular cohomology and/or intersection cohomology of algebraic varieties to better understand representation theory. To that end, we will discuss some sheaf theory, focus on perverse sheaves on flag varieties, and relate this example to representations of semisimple Lie algebras.

Approximate Course Outline

- (3 weeks) categorical catch-up: additive, abelian, and triangulated categories, and functors between them; t-structures; derived categories.
- (4 weeks) sheaf theory: sheaves and functors, locally constant and constructible sheaves, Verdier duality, gluing, the perverse t-structure.
- (4 weeks) perverse sheaves on flag varieties: the flag variety, Bruhat decomposition, highest weight categories, projective cover of the skyscraper sheaf and its properties, convolution.
- (3 weeks) Mixed sheaves and purity; graded versions of categories. Koszul Duality.
- (1 week) Depending on interests of class; possibly seminar style introductions to some other geometric representation theory topics, for instance (1) parity sheaves (2) geometric Satake equivalence (3) Springer correspondence.

Homework Problem sets will be due approximately every two weeks. Each problem set will consist of about 3-5 problems. The letter grade will be based on the number of problem sets submitted with substantial work. Under normal circumstances, I expect everyone to earn an ‘A’. If you feel that you are getting behind, please see me as soon as possible.

References There is no required textbook. Initially we will follow some course notes authored by P. Achar. I will provide a link (by email). The following may also be useful:

C. Weibel, *An introduction to homological algebra*.

A. Beilinson, V. Ginzburg, and W. Soergel, *Koszul duality patterns in representation theory*, J. Amer. Math. Soc. 9 (1996), no. 2, 473–527.

J. Humphreys, *Representations of Semisimple Lie algebras in BGG Category \mathcal{O}* .

Academic Honesty As a University of Georgia student, you have agreed to abide by the University’s academic honesty policy, *A Culture of Honesty*, and the Student Honor Code. All academic work must meet the standards described in *A Culture of Honesty* found at: <https://ovpi.uga.edu/academic-honesty/academic-honesty-policy>. Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Questions related to course assignments and the academic honesty policy should be directed to the instructor.

Disclaimer The course syllabus is a general plan for the course; deviations announced to the class by the instructor may be necessary.