1. Show the morphism $\mathbb{Z} \subset \mathbb{Q}$ is epi in the category of (associative) rings. (Rings are not assumed to have multiplicative identity).

2. Construct an example in the category of abelian groups to show $\mathrm{Hom}(A, -)$ is not in general exact.

3. Assume R is a ring. Which of the following additive categories are abelian? (You may use that $\mathrm{mod}R$ is known to be abelian.) If not abelian in general, give an assumption to guarantee abelian. (a) $\mathrm{Proj}R$ (the category of projective R-modules) (b) $\mathrm{fgmod}R$ (the category of finitely generated R modules) (c) abelian p-groups (d) $\mathcal{A}^C$ (the category of functors $C \to \mathcal{A}$, where $\mathcal{A}$ is abelian and C any category).

4. Let Vect denote the category of vector spaces. Define shift $[1] : \mathrm{Vect} \to \mathrm{Vect}$ to be the identity functor, and call a sequence $X \to Y \to Z \to X[1]$ distinguished if it is exact at X, Y, and Z. Show this gives Vect the structure of a triangulated category.

5. Let $\mathcal{A}$ be an abelian category. Try the same approach as in the previous exercise to put a triangulated structure on $\mathcal{A}$. Show this works (i.e. all triangulated category axioms hold) if and only if $\mathcal{A}$ is semisimple.

6. Let $\mathcal{A}$ denote an abelian category. (You're welcome to work directly in the category of abelian groups.) Show a homotopy equivalence in $\mathrm{Ch}(\mathcal{A})$ is a quasi-isomorphism.

7. Show that quasi-isomorphisms are not necessarily homotopy equivalences by considering an example in $\mathrm{Ch}(\mathrm{mod}\mathbb{C}[x])$. That is, show $0 \to \mathbb{C}[x] \xrightarrow{x} \mathbb{C}[x] \to 0$ vs $0 \to 0 \to \mathbb{C} \to 0$ are quasi-isomorphic, but not homotopy equivalent. (Here: $\mathbb{C}$ is understood to be the $\mathbb{C}[x]$ module with x acting by 0.)