1. If a population P(t) is known to grow exponentially, P(0) = 10,000, and P(3) = 80,000, what is P(7)?

2. Solve the initial value problem using either an integrating factor or with guessing or using Laplace transform
$$\frac{dy}{dt} + 2y = 3e^{-2t}, \qquad y(0) = 1.$$

3. Find the solution of the initial value problem with or without using the Laplace transform
$$\frac{dy}{dt} = \frac{V(t) - y}{RC}, \qquad y(0) = 1,$$
where $R = 0.5, C = 1.0$, and $V(t)$ is the function that is constantly 0 for $t < 3$ and 1 for $t \geq 3$.

4. Consider the population model for a species of fish in a lake.
$$\frac{dP}{dt} = 2P - \frac{P^2}{50}$$
Fishing will be allowed, but it is unclear how many fishing licenses should be issued. If the average catch of a licensed fisherman is 3 fish per year, what is the largest number of licenses that can be issued if the fish are to have a chance to survive in the lake? You should justify that the fish population will survive with this many licenses issued, but will not survive if more licenses are issued.

5. A 50 gallon tank of sugar solution begins at time zero with a sugar concentration of 1/2 cup per gallon. Sugar solution with concentration 2 cups per gallon is added to the tank at a rate of 2 gallons per minute while, at the same time, 2 gallons per minute of the mixed solution is drawn from the tank. (a) What is the initial value problem modeling the total amount of sugar in the tank at time t?

   (b) What is the concentration of sugar in the tank after 10 minutes? What value will the concentration of sugar in the tank approach after a long time?

6. Consider the differential equation $\frac{dy}{dt} = -y^2 + 7y - 10$.

   (a) Sketch the phase line for the differential equation. Identify equilibrium points as sinks, sources, or nodes.

   (b) Sketch graphs of solutions satisfying the initial conditions $y(0) = 0$ and $y(0) = 3$. Put all graphs on one pair of axes.

   (c) Let $y(t)$ be the solution to the initial value problem $\frac{dy}{dt} = -y^2 + 7y - 10, y(0) = 3$. Approximate $y(1)$ using Euler's method with step size $\Delta t = 1$.

7. Consider the following linear system
$$\frac{dY}{dt} = \begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix} Y, \qquad \text{where } Y = \begin{pmatrix} x \\ y \end{pmatrix}.$$

   (a) What is the general solution to the linear system?

   (b) Sketch the phase portrait for the system.

8. Solve the initial value problem with or without using the Laplace transform.
$$\frac{d^2y}{dt^2} + 9y = 3\cos(2t), \quad y(0) = 0, \quad y'(0) = 3$$

9. Suppose the 2×2 matrix A has eigenvalue $\lambda = 2 + 4i$ with eigenvector $\begin{pmatrix} 1 \\ i \end{pmatrix}$.

   (a) What is the general solution to the linear system $\dfrac{dY}{dt} = AY$?

   (b) Solve the initial value problem $\dfrac{dY}{dt} = AY, \quad Y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

   (c) Classify the system given (Type: sink, source, saddle, spiral sink, spiral source, or periodic).

10. (a) Classify each linear system given below (Type: sink, source, saddle, spiral sink, spiral source, or periodic). When appropriate, determine whether solutions travel clockwise or counterclockwise about the origin (Direction). Give justification for each. Do not solve!

(a) $\dfrac{dY}{dt} = \begin{pmatrix} 0 & -5 \\ 1 & 0 \end{pmatrix} Y$.	Type:	Direction:
(b) $\dfrac{dY}{dt} = \begin{pmatrix} -5 & -9 \\ 3 & 1 \end{pmatrix} Y$.	Type:	Direction:
(d) $\dfrac{dY}{dt} = \begin{pmatrix} -5 & -6 \\ -2 & -5 \end{pmatrix} Y$.	Type:	Direction:

11. Assume a spring is modeled by the equation $\dfrac{d^2y}{dt^2} + b\dfrac{dy}{dt} + 25y = 0$ with $b > 0$ a constant. Find a value for b so that for any larger value of b, the spring does not oscillate, but for any smaller (but positive) value of b, the spring does oscillate.