1. Prove if $a, b \in \mathbb{Z}_n$ and a is a unit, then $ax = b$ has a unique solution.
2. Find multiplicative inverse of a unit a in $\mathbb{Z}_n$. Example: $a = 4$ and $n = 27$ or $a = 6$ and $n = 49$.
3. (3.1 #35)Prove or disprove: (1) If R and S are integral domains, then $R \times S$ is an integral domain. (2) If R and S are field , then $R \times S$ is a field.
4. (3.1 #38) Let R be a ring, and fix $a \in R$. Then the **right annihilator** of a in R is given by the set $A_R = \{r \in R | ar = 0_R\}$. Prove A_R is a subring of R. Is it an ideal? Left/right ideal?
5. (3.3 #15) Let $f : R \to S$ be a ring homomorphism. If r is a zero divisor in R, is $f(r)$ a zero divisor in S?
6. (4.1 #5) Apply division algorithm to find $q(x)$ and $r(x)$ with $r(x) = 0$ or $\deg(r(x)) < \deg(g(x))$.
   (1) $f(x) = 3x^4 - 2x^3 + 6x^2 - x + 2$ and $g(x) = x^2 + x + 1$ in $\mathbb{Q}[x]$.
   (2) $f(x) = x^4 - 7x + 1$ and $g(x) = 2x^2 + 1$ in $\mathbb{Q}[x]$.
   (3) $f(x) = 2x^4 + x^2 - x + 1$ and $g(x) = 2x - 1$ in $\mathbb{Z}_5[x]$.
7. (4.3 #10) Is the given polynomial irreducible:
   (1) $x^2 - 3$ in $\mathbb{Q}[x]$? (You should justify why $\sqrt{3}$ is not rational.) in $\mathbb{R}[x]$?
   (2) $x^2 + x - 2$ in $\mathbb{Z}_3[x]$? In $\mathbb{Z}_7[x]$?
8. (5.2 #5, 9, 14) See attached.
9. Consider $\mathbb{Q}(\sqrt{2}) = \{r + s\sqrt{2} | r, s \in \mathbb{Q}\}$. This is a subfield in $\mathbb{R}$. (Check.) Show $\mathbb{Q}(\sqrt{2})$ is isomorphic to $\mathbb{Q}[x]/(x^2 - 2)$ by applying the First Isomorphism Theorem to the "evaluate x at $\sqrt{2}$" map $f : \mathbb{Q}[x] \to \mathbb{Q}(\sqrt{2})$. You should check that f is a surjective ring homomorphism. What is its kernel?
10. (6.2 #4) Let $[a]_n$ denote the congruence class of the integer a modulo n.
    (1) Show that the map $f : \mathbb{Z}_{12} \to \mathbb{Z}_4$ given by $[a]_{12} \mapsto [a]_4$ is a well-defined, surjective ring homomorphism. What is it's kernel? Apply the First Isomorphism Theorem.
11. (6.2 #5)Let I be an ideal in an integral domain R. Is it true that R/I is an integral domain?
12. (6.2 #21) Use the First Isomorphism Theorem to show that $\mathbb{Z}_{20}/(5) \cong \mathbb{Z}_5$.
13. (6.3 #5) List all maximal ideals in $\mathbb{Z}_6$. Same for $\mathbb{Z}_{12}$.
14. (11.1 #4) Fix an integer $n \geq 1$. Show the set of all polynomials in $\mathbb{R}[x]$ with degree less than or equal to n is an $\mathbb{R}$ vector space. (Is this subset also a subring?)
15. (11.1 #23) Show $\{1, \sqrt{2}, \sqrt{3}\}$ is linearly independent over $\mathbb{Q}$.
16. (11.1 #24) Let v be a nonzero real number. Prove $\{1, v\}$ is linearly independent over $\mathbb{Q}$ if and only if v is irrational.
17. (11.1 #25) Show $\{1, x, x^2, ..., x^k\}$ is $\mathbb{R}$-linearly independent in $\mathbb{R}[x]$ for integer $k \geq 1$.