MATH 2250 - Royal Section 2.6 - Limits Involving Infinity; Asymptotes of Graphs

Note: If you know it, you may NOT use L'Hopital's rule.

1. Use a table to estimate the value of $\lim_{x\to 3^-} \dfrac{1}{x-3}$.

x	$f(x)$
2.9	
2.99	
2.999	
2.9999	

reasoning:

What about $\lim_{x\to 3^+} \dfrac{1}{x-3}$?

2. Determine $\lim_{x\to \pi^+} \csc(x)$.

Definition. The vertical line $x = a$ is a vertical asymptote to the graph of f if at least one of these is true:

$$\lim_{x\to a} f(x) = \infty \text{ or } -\infty$$
$$\lim_{x\to a^-} f(x) = \infty \text{ or } -\infty$$
$$\lim_{x\to a^+} f(x) = \infty \text{ or } -\infty$$

Rule of Thumb. If you are looking at a *quotient* of two polynomial functions and you want to find *potential* vertical asymptotes, look for numbers that make the denominator function equal zero.

3. Determine all vertical asymptotes of the function $f(x) = \dfrac{1}{x-3}$.

4. Determine all vertical asymptotes of the function $f(t) = \dfrac{36-t}{6-\sqrt{t}}$. Use limits to support your answer.

5. Determine all vertical asymptotes of $y = \ln(x)$.

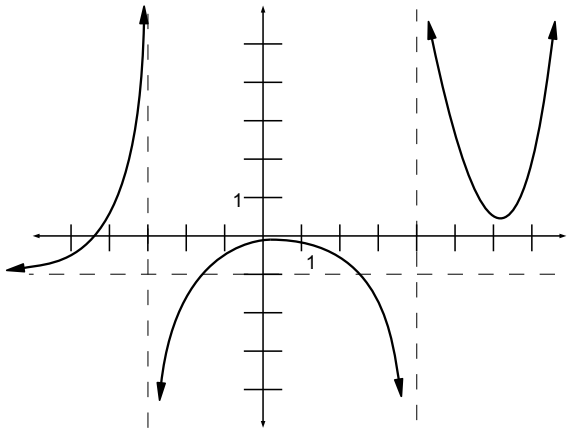
Important fact. If $r > 0$ is a rational number, then $\lim_{x\to\infty} \frac{1}{x^r} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^r} = 0$ (provided that x^r is defined).

6. (a) Determine $\lim_{x\to\infty} \frac{2x^2 - 3x + 1}{x^2 + 5}$.

   (b) In terms of the graph of $f(x) = \frac{2x^2 - 3x + 1}{x^2 + 5}$, what does our answer mean?

Definition. The horizontal line $y = b$ is a horizontal asymptote to the graph of f if at least one of these is true: $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

7. The graph of $y = f(x)$ is below. Assume that all pertinent information is shown in the graph, and that the domain of f extends infinitely far in both directions. Determine all vertical and horizontal asymptotes of f. Use limits to support your answer.



8. Determine all *horizontal* asymptotes of $f(x) = \frac{1}{x - 3}$. Use limits to support your answer.

9. Determine the following limits.

   (a) $\lim_{x\to-\infty} (-x^5 + x^4 - 1)$

   (b) $\lim_{x\to-\infty} \frac{-2x + 5}{4x + 7}$

   (c) $\lim_{x\to\infty} \frac{-7x^4 + 5x}{1 + x}$

1. (a) Determine $\lim_{x\to\infty} \dfrac{2x^3+3x+1}{1-x^2}$.

   (b) Use division of polynomials to rewrite the expression $\dfrac{2x^3+3x+1}{1-x^2}$.

   (c) What does your answer to the previous part tell you about the behavior of $f(x) = \dfrac{2x^3+3x+1}{1-x^2}$ when x is a very large positive number?

Definition. For a rational function of two polynomials, when the numerator's degree is one more than the denominator's degree, we say that the function has a slant asymptote (or oblique asymptote).

2. Determine all asymptotes of the function $f(x) = \dfrac{2x^3+3x+1}{1-x^2}$.

3. Determine $\lim_{x\to\infty} \dfrac{3e^{-x}}{x+27}$.

4. Determine $\lim_{x\to\infty} \dfrac{2e^x+3e^{-x}}{e^x-4e^{-x}}$.

5. Determine $\lim_{x\to-\infty} \dfrac{2e^x+3e^{-x}}{e^x-4e^{-x}}$.

Important Facts:

6. Determine $\lim_{x\to\infty} \dfrac{3|x| - 4x}{2|x| + 5x}$.

7. Determine $\lim_{x\to-\infty} \dfrac{3|x| - 4x}{2|x| + 5x}$.

8. $\lim_{t\to-\infty} \dfrac{2t}{\sqrt{t^2 + 3t}}$

9. $\lim_{x\to\infty} \sqrt{9x^2 + x} - 3x$

Important Facts:

10. Determine $\lim_{x\to\infty} \dfrac{(x-1)(\cos(x))}{x^2}$.