

From the 2.1 Activity, for the function  $f(x) = x^2$ : it looks like  $m_{tan}(a)$  should be  $2a$ , regardless of which number  $a$  we are looking at.

Definition. The derivative of  $f(x)$ , denoted by  $f'(x)$ , is the function defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

wherever that limit exists.

Note. To determine something like  $f'(3)$ , we could use 3.1 methods *or* we could determine  $f'(x)$  using our new formula, and then plug in 3 for  $x$  at the end.

Terminology. Determining the derivative is called differentiating or differentiation.

Alphabet soup. These mean the same thing:  $f'(x)$ ,  $f'$ ,  $\frac{d}{dx}(f)$ ,  $\frac{dy}{dx}$ ,  $y'$ ,  $\frac{df}{dx}$

1. Use the *definition of the derivative* to determine the derivative of  $f(x) = x^2$ .
2. Use the definition of the derivative to determine the derivative of  $g(x) = \sqrt{2+x}$ .
3. Use the definition of the derivative to determine the derivative of  $y = \frac{x}{3-x}$ . Then determine an equation of the tangent line to the graph of  $f$  at  $(2, 1)$ .

Definition. If  $f'(a)$  exists, we say that  $f$  is differentiable at  $a$ .

4. Determine where  $f(x) = x^2$  is differentiable.

5. Determine where  $g(x) = \sqrt{2+x}$  is differentiable.

Theorem. If  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ .

6. Determine intervals where  $g(x) = \sqrt{2+x}$  is *continuous*.

Caution. A function might be continuous at a number but not differentiable at that same number!!

7. Determine intervals where  $f(x) = |x|$  is continuous. Determine the derivative of  $f(x) = |x|$ , and determine intervals where  $f(x)$  is differentiable.

8. Use the definition of the derivative to differentiate  $y = \frac{2}{\sqrt{1-7x}}$ .