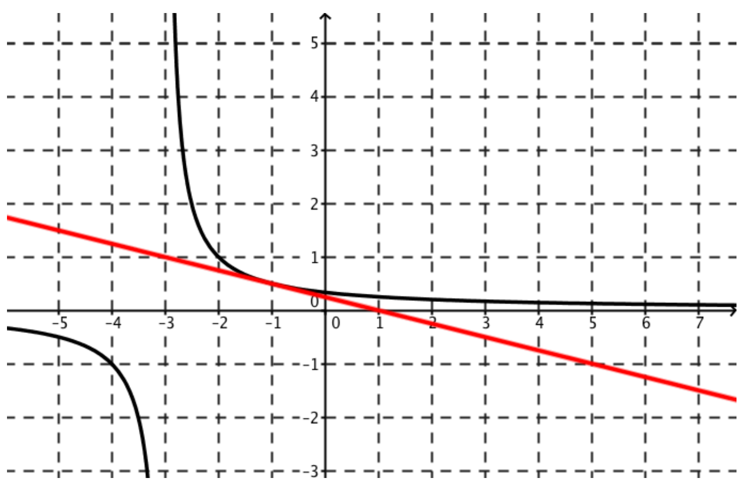
Recap. Where does the tangent line come from?

- secant line ($h \neq 0$):
- formula for slope of the secant line ($h \neq 0$):
- Imagine moving $(a+h, f(a+h))$ closer and closer to $(a, f(a))$. What happens to the secant line?
- formula for slope of the tangent line at $(a, f(a))$:

Important Idea. slope of the tangent line = rate of change of y with respect to x

- velocity is the rate of change of position (y) with respect to time (x) (e.g. for a car moving along a straight road)
- rate of growth of bacteria (y) with respect to time (x) - topic from biology
- rate of change of profit (y) with respect to level of sales (x) - topic in economics

1. Below are the graphs of $f(x) = \dfrac{1}{x+3}$ and the tangent line to the graph of f at the point $(-1, \frac{1}{2})$. Estimate the rate of change of y with respect to x at $x = -1$ using *only* the tangent line to the graph of f.



Definition. Let $(a, f(a))$ be a point on the graph of f. Then the tangent line at $(a, f(a))$ to the graph of f at the point $(a, f(a))$ has slope

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If this limit doesn't exist at $(a, f(a))$, then there is no tangent line to the graph at that point. Other names for m_{tan} include $f'(a)$, $y'(a)$, and $\left.\dfrac{dy}{dx}\right|_{x=a}$.

2. Use the formula for m_{tan} to determine the slope of the tangent line to the graph of $f(x) = \dfrac{1}{x+3}$ at the point $(-1, \frac{1}{2})$. Then determine an equation of the tangent line to the graph at that point.

3. Use the formula for m_{tan} to determine $f'(-2)$ for the function $f(x) = x^3 - x$.

Recall from 2.1 that the average rate of change of the function f over the interval $[a, a+h]$ is $[f(a+h) - f(a)]/h$. Using our limit notation, the instantaneous rate of change of a function f with respect to its input variable at a is m_{tan} (if it exists).

Note: The instantaneous rate of change of *the position function* with respect to time is called velocity.

4. Suppose a particle is moving along the x-axis, and its position at time t is $f(t) = \sqrt{t-5}$. Use our formula for m_{tan} to determine the velocity of the particle with respect to time when $t = 6$.

5. Use the formula for m_{tan} to determine $f'(2)$ for the function $f(x) = \dfrac{4x}{3-x}$.