MATH 3200 - Royal
PRACTICE Exam # 3
Fall 2019

Name (Print): ______________________________

Friendly reminders:

- You may not leave the room during the exam period. (If you have a medical issue, let me know.)
- Smart watches, cell phones, and other personal electronic devices must not be on your person and must be stored.
- You don't have to work through the test in order. Go in the order you want to.
- I hope you do a great job!

| Problem | Score | Out of |
|---|---|---|
| 1 | | 25 |
| 2 | | 25 |
| 3 | | 15 |
| 4 | | 15 |
| 5 | | 20 |
| Total | | 100 |

**I will be academically honest in all my academic work and will not tolerate academic dishonesty of others.**

Signed: ______________________________________________ Date: ____________

1. (5 points each part)

   (a) Let $\sim$ be a relation on a set $X$. Define: $\sim$ is symmetric.

   (b) Let $\sim$ be a relation on a set $X$. Define: $\sim$ is transitive.

   (c) Define: $x$ is a rational number.

   (d) Suppose $f$ is a relation on $X$ and $Y$. Define: $f$ is a function.

   (e) Suppose $f : X \to Y$ is a function and $B \subseteq Y$. Define: the preimage of $B$.

2. (5 points each) For this problem, refer to the set $S = \{a, b, c\}$.

   (a) Give an example of an equivalence relation on $S$.

   (b) Give an example of a relation on $S$ that is reflexive and is symmetric, but is not transitive, or explain why it is impossible to give such a relation.

   (c) Give an example of a relation on $S$ that is not reflexive, but is symmetric and is transitive, or explain why it is impossible to give such a relation.

   (d) The relation $\mathcal{R} = \{(a, b), (b, a)\}$ is a relation on $S$. Is it symmetric? Explain briefly.

   (e) The relation $\mathcal{T} = \{(a, a)\}$ is a relation on $S$. Is it an equivalence relation? Explain briefly.

3. (15 points) Let $A$ and $B$ be sets. Use Axiom 4.24 to prove that

$$(A \cap B)^c = A^c \cup B^c$$

4. (15 points) Let $B$ be a set, and let $S = \mathcal{P}(B)$, the power set of $B$. Define a relation $\sim$ on $S$ by $X \sim Y$ if $X \subseteq Y$. Prove that $\sim$ is a partial ordering on $S$.

5. (20 points) Let $S = \mathbb{Z}$, and define the relation $\sim$ on $S$ by $a \sim b$ if $3|(a-b)$. Show that $\sim$ is an equivalence relation.

Note: This relation is very important in MATH 4000/6000. We say that $a \cong b \pmod{n}$ (read aloud as "$a$ is congruent to $b$ mod $n$") if $n|(a-b)$. In this problem, you are proving that congruence mod 3 is an equivalence relation.

Here is an extra page for scratch work. Make a note on the problem page if you want me to look at your work here.