

MATH 3200 - Royal  
Practice Exam # 2

Name (Print): \_\_\_\_\_

Friendly reminders:

- Smart watches, cell phones, and other personal electronic devices must not be on your person and must be stored.
- You don't have to work through the test in order. Go in the order you want to.
- I hope you do a great job!

Problem	Score	Out of
1		25
2		15
3		20
4		20
5		20
Total		100

**I will be academically honest in all my academic work and will not tolerate academic dishonesty of others.**

Signed: \_\_\_\_\_ Date: \_\_\_\_\_

1. Consider the sets  $A = \{1, 2, 3, 4\}$  and  $B = \{2, 7\}$ . Determine the following (no explanation is required):

(a) (5 points)  $A \cup B$

(b) (5 points)  $\mathcal{P}(B)$

(c) (5 points)  $B \setminus A$

(d) (5 points)  $(B \setminus A) \times B$

(e) (5 points)  $A \cap \{\{1\}, 2\}$

2. (15 points) Prove the Transitive Property for Subsets: Let  $A$ ,  $B$ , and  $C$  be sets. Prove that if  $A \subseteq B$  and  $B \subseteq C$ , then  $A \subseteq C$ .

3. (a) (10 points) Let  $A$ ,  $B$  and  $X$  be sets. Prove that if  $X \subseteq (A \cap B)$ , then  $X \subseteq A$  and  $X \subseteq B$ . You may only use definitions in your proof.

- (b) (10 points) Let  $A$ ,  $B$  and  $X$  be sets. Use an example to disprove this statement:  
if  $X \subseteq (A \cup B)$ , then  $X \subseteq A$  or  $X \subseteq B$ .

Your work must clearly show that the statement is false, but you do not have to use complete sentences in your work.

4. (20 points) Prove that, for every natural number  $n$ ,

$$(-1)^1 (1^2) + (-1)^2 (2^2) + (-1)^3 (3^2) + \cdots + (-1)^n (n^2) = (-1)^n \left( \frac{n(n+1)}{2} \right)$$

5. (20 points) Define the sequence  $a_n$  recursively as follows:  $a_1 = 1$ ,  $a_2 = 1$ , and for  $n \geq 3$ ,  $a_n = \frac{1}{a_{n-1} + a_{n-2}}$ . Prove for all  $n \in \mathbb{N}$  that

$$\frac{1}{2} \leq a_n \leq 1.$$

(Hint: You may use any valid properties of inequalities that you know from previous math experience.)

Here is an extra page for scratch work. Make a note on the problem page if you want me to look at your work here.