MATH 3200 - Royal
Exam # 2
Summer 2019

Name (Print): _____

Friendly reminders:

- You may not leave the room during the exam period.
- Smart watches, cell phones, and other personal electronic devices must not be on your person and must be stored.
- You don't have to work through the test in order. Go in the order you want to.
- I hope you do a great job!

Problem	Score	Out of
1		30
2		15
3		15
4		20
5		20
Total		100

I will be academically honest in all my academic work and will not tolerate academic dishonesty of others.

Signed: _____ Date: _____

1. Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{2, 7\}$. Determine the following (no explanation is required):

   (a) (5 points) $A \cup B$

   (b) (5 points) $\mathcal{P}(B)$

   (c) (5 points) $\mathcal{P}(B) \setminus \{\{2\}, 7\}$

   (d) (5 points) $B \setminus A$

   (e) (5 points) $(B \setminus A) \times B$

   (f) (5 points) $A \cap \{\{1\}, 2\}$

2. (15 points) Let A, B, and C be sets. Use the setbuilder method to prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

Be sure to reference every definition or property you are using; you may not use any results previously proved about sets. (In particular, you may not assume the statement you are trying to prove.)

3. (15 points) For sets A, B, and C, use set properties to prove that

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

In your proof, remember to only use one property at a time.

4. (20 points) Prove that, for every natural number n,

$$(-1)^1\left(1^2\right)+(-1)^2\left(2^2\right)+(-1)^3\left(3^2\right)+\cdots+(-1)^n\left(n^2\right)=(-1)^n\left(\frac{n(n+1)}{2}\right)$$

5. (a) (10 points) Let A, B and X be sets. Prove that if $X \subseteq (A \cap B)$, then $X \subseteq A$ and $X \subseteq B$. You may only use definitions in your proof.

(b) (10 points) Let A, B and X be sets. Use an example to disprove this statement:

$$\text{if } X \subseteq (A \cup B), \text{ then } X \subseteq A \text{ or } X \subseteq B.$$

Your work must clearly show that the statement is false, but you do not have to use complete sentences in your work.

Here is an extra page for scratch work. Make a note on the problem page if you want me to look at your work here.