

Friendly reminders:

- You may not leave the room during the exam period.
- Smart watches, cell phones, and other personal electronic devices must not be on your person and must be stored.
- You don't have to work through the test in order. Go in the order you want to.
- I hope you do a great job!

Problem	Score	Out of
1		30
2		10
3		20
4		20
5		20
Total		100

Rules for inequalities.
Assume a , b , and c are real numbers.
(Rule 1) If $a > b$ and $b > c$, then $a > c$.
(Rule 2) If $a > b$ then $a + c > b + c$.
(Rule 3) If $a > b$ and $c > 0$, then $ac > bc$.
(Rule 4) If $a > b$ and $c < 0$, then $ac < bc$.
(Rule 5) For every real number a , exactly one of the following is true:
$a > 0$ $a = 0$ $a < 0$

I will be academically honest in all my academic work and will not tolerate academic dishonesty of others.

Signed: _____ Date: _____

1. Determine the truth value of each statement. Then negate each statement; write your negations using the simplest language possible. (You do not have to explain.)

(a) (10 points) Statement: For any positive integer z , the number \sqrt{z} is an integer.

Truth Value:

Negation of Original Statement:

(b) (10 points) Statement: There exists a real number x satisfying $x^2 - 2 = 0$.

Truth Value:

Negation of Original Statement:

(c) (10 points) Statement: If z is an integer, then $z = 0$ or $z^2 > 1$.

Truth Value:

Negation of Original Statement:

2. (10 points) Use a truth table to prove that $(A \vee B) \wedge C$ is logically equivalent to $(A \wedge C) \vee (B \wedge C)$.

3. (20 points) **Note.** For this problem, you may only use the definitions of even integer and odd integer: you may not use any results previously proved about even and odd integers.

Prove: Let n be an integer. Then $n^2 - 2$ is odd if and only if n is odd.

4. (20 points) **Note.** For this problem, you may only use the definitions of even integer and odd integer: you may not use any results previously proved about even and odd integers.

Prove: Let x and y be integers. If x is odd and y is even, then $3x + 5y$ is odd.

5. (20 points) **Note.** For this problem, you may only use the rules for inequalities that are given below. You may not use any results previously proved about inequalities, or any results that are familiar to you from past work with inequalities.

Rules for inequalities.
Assume a , b , and c are real numbers. (Rule 1) If $a > b$ and $b > c$, then $a > c$. (Rule 2) If $a > b$ then $a + c > b + c$. (Rule 3) If $a > b$ and $c > 0$, then $ac > bc$. (Rule 4) If $a > b$ and $c < 0$, then $ac < bc$. (Rule 5) For every real number a , exactly one of the following is true: $a > 0$ $a = 0$ $a < 0$

Prove: Let x, y, z , and w be positive real numbers. If $x > y$ and $z > w$, then $xz > yw$.

Here is an extra page for scratch work. Make a note on the problem page if you want me to look at your work here.