

Differentiation Rules

MATH 2250 - Dr. Royal

For all rules below, assume that f and g are differentiable functions and a, b, c are constants.

The Constant Rule The derivative of a constant is zero:

$$\frac{d}{dx}(c) = 0.$$

The Power Rule If n is any real number, then

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}.$$

The Constant Multiple Rule The derivative of a constant times a function = the constant times the derivative of the function:

$$\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x)).$$

The Sum Rule Taking the derivative of two functions that are added together = taking the derivative of each function and adding the answers:

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)).$$

The Difference Rule Taking the derivative of the difference of two functions = taking the derivative of each function and subtracting:

$$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x)).$$

Derivatives of Exponential Functions If $a > 0$, then $\frac{d}{dx}(a^x) = a^x \ln(a)$

Differentiation Rule for e^x

$$\frac{d}{dx}(e^x) = e^x$$

The Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f(x) \frac{d}{dx}(g(x)) + g(x) \frac{d}{dx}(f(x))$$

The Quotient Rule If $g(x) \neq 0$ then

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx}(f(x)) - f(x) \frac{d}{dx}(g(x))}{(g(x))^2}.$$

Derivatives of Trig Functions

$$\begin{aligned} \frac{d}{dx}(\sin(x)) &= \cos(x) & \frac{d}{dx}(\cos(x)) &= -\sin(x) & \frac{d}{dx}(\tan(x)) &= \sec^2(x) \\ \frac{d}{dx}(\sec(x)) &= \sec(x) \tan(x) & \frac{d}{dx}(\csc(x)) &= -\csc(x) \cot(x) & \frac{d}{dx}(\cot(x)) &= -\csc^2(x) \end{aligned}$$

The Chain Rule If $h(x) = f(g(x))$, then $h'(x) = f'(g(x)) \cdot g'(x)$.

In Leibniz notation, if $h = f(u)$ and $u = g(x)$, then

$$\frac{dh}{dx} = \frac{dh}{du} \cdot \frac{du}{dx}.$$

Differentiating a Function to a Power If $u = g(x)$ and n is any real number, then

$$\frac{d}{dx}(u^n) = n \cdot u^{n-1} \cdot \frac{du}{dx}.$$

Differentiating e to a Function If $u = g(x)$, then $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$.

Derivatives of Logarithmic Functions

$$\begin{aligned} \frac{d}{dx}(\ln(x)) &= \frac{1}{x} \\ \frac{d}{dx}(\log_a(x)) &= \frac{1}{x \ln(a)} \end{aligned}$$

Differentiating the Natural Log of a Function If $u = g(x)$, then

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u} \cdot \frac{du}{dx}.$$

Derivatives of Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$