Definition. Let f be a function defined on an open interval containing all values of x close to a. Then f is continuous at a if $\lim_{x \to a} f(x) = f(a)$.

Continuity Checklist for a function $f(x)$ defined on an open interval around a.

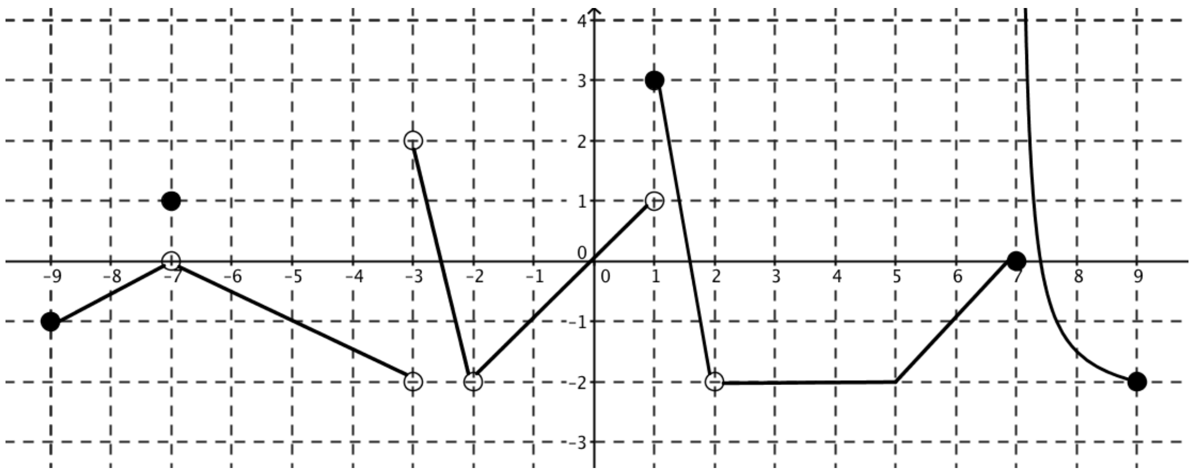
1. Is $f(a)$ defined?
2. Does $\lim_{x \to a} f(x)$ exist?
3. Is $f(a)$ equal to $\lim_{x \to a} f(x)$?

Note. If the answer to all three is "yes," then f is *continuous* at a. If the answer to at least one of the above questions is "no," then we say that f is *discontinuous* at a or f has a *discontinuity* at a.

Examples. Be really careful about how you write these down. You will have to write correct mathematics to get full credit.

1. Is $f(x) = x^2 - 5x + 2$ continuous at $x = 3$? Use the three-item continuity checklist to justify your answer.

2. Determine whether the function $f(x)$ whose graph is below is continuous at the following values of x. Use the three-item continuity checklist to justify your answers. Also identify the type of each discontinuity.



$x = 1$

$x = -5$

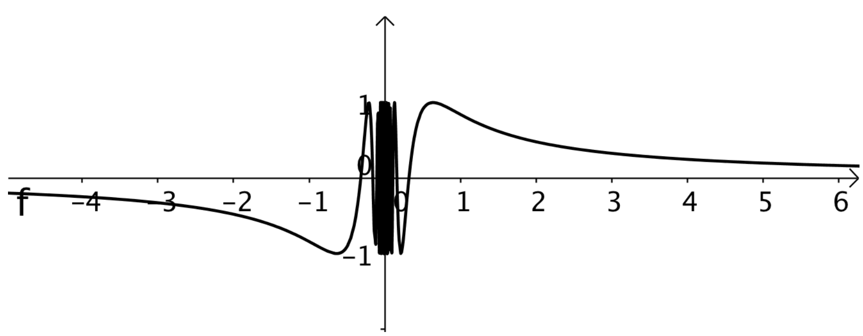
$x = -7$

$x = 7$

3. Find the numbers, if any, where the function $f(x) = \sec(3x)$ is discontinuous.

4. Consider the function $f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 3 - x^2 & \text{if } x \geq 1 \end{cases}$. Is f continuous at $x = 1$? Use the three-item continuity checklist to justify your answer. If f is not continuous, identify the type of discontinuity.

5. Consider the function $f(x) = \sin(1/x)$ whose graph is below. Determine where the function is continuous.



Continuity from the right and left. Let f be a function defined on an interval $[a, b]$. Then

- f is *continuous from the right* at a if $\lim_{x \to a^+} f(x) = f(a)$

- f is *continuous from the left* at b if $\lim_{x \to b^-} f(x) = f(b)$.

Continuity on various intervals.

- f is continuous on (a, b) if it is continuous at every number in (a, b).

- f is continuous on $[a, b]$ if it is continuous on (a, b) and cts from the right at a and cts from the left at b

- f is continuous on $[a, b)$ if it is continuous on (a, b) and cts from the right at a

- f is continuous on $(a, b]$ if it is continuous on (a, b) and cts from the left at b

5. Determine intervals where the function is continuous.

   (a) $f(x) = x^2 - 5x + 2$

   (b) $f(x) = \sqrt{1 - x}$

Types of functions that are continuous The following functions are continuous *on their domains*: polynomials, rational functions, root functions, trig functions, inverse trig functions.

Building continuous functions from other functions. If f and g are continuous at a, so are $f+g$, $f-g$, fg, cf (where c is a constant), and $\frac{f}{g}$, as long as $g(a) \neq 0$.
Also. Compositions of continuous functions are continuous.

The Intermediate Value Theorem (IVT)
If f is continuous on the closed interval $[a, b]$ and M is any number between $f(a)$ and $f(b)$, inclusive, then there is at least one number c in $[a, b]$ such that $f(c) = M$.

6. Prove that the equation $x^5 + 17x - 10 = 0$ has a solution in the interval $[0, 1]$.