Motivating Example. Determine $\int (x^5 - 4x + 7)^9 (5x^4 - 4)\ dx$.

The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du = F(u) + C = F(g(x)) + C,$$

where F is an antiderivative of f.

Steps for determining an integral using this rule:

1. Try to match up the parts of your integrand with the parts in "Take the derivative of the outside function, leaving the inside function alone. Then multiply by the derivative of the inside function."
2. Let $u = g(x)$, where $g(x)$ is usually an "inside function" in a composition.
3. Determine $g'(x)$ and *find* it inside your integrand, and write $du = g'(x)dx$.
4. Sub in $u = g(x)$ and $du = g'(x)dx$ to get an indefinite integral with u's in it but no x's in it: $\int f(u)\ du$.
5. Integrate.
6. Sub the x's back in and get rid of all of the u's.

Exercises. Evaluate the following indefinite integrals.

1. $\int 3\sin(3x)\ dx$

2. $\int \frac{x^2}{1+x^3}\ dx$

3. $$\int \frac{1+\frac{1}{x}}{(x+\ln(x))^3}\ dx$$

4. $$\int \frac{-5e^x}{\sqrt{1-e^{2x}}}\ dx$$

5. $$\int \cos^2(x)\ dx$$

6. $$\int 5\cot(4u)\ du$$

7. $$\int x^2(1-x)^7\ dx$$

8. $$\int \sec(x)\ dx$$

Determine the following indefinite integrals.

1. $\int (7x+3)^{-1/2} dx$

2. $\int \frac{3x^2-5}{8-5x+x^3} dx$

3. $\int \frac{\arccos(x)}{\sqrt{1-x^2}} dx$

4. $\int x^2 e^{7-x^3} dx$

5. $\int \frac{\cos(\ln(x))}{x}\, dx$

6. $\int \left(-\sin(x)+\sin^3(x)\right)\, dx$

7. $$\int \frac{\sqrt[4]{\log_3(x)}}{7x}\ dx$$

8. $$\int \frac{\cos(x)}{1+\sin^2(x)}dx$$

9. $$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$$

10. $$\int \tan(x)dx$$

11. $$\int \frac{(1+\arctan(x))^6}{1+x^2}dx$$

12. $$\int \frac{1}{1+49x^2}dx$$