Motivating Example. $\displaystyle\int_a^b x\,dx$

| The Fundamental Theorem of Calculus, Part 2 |
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| If $f$ is continuous on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F$ is *any* antiderivative of $f$. |

FTC Part 2 works because of the Mean Value Theorem: If $F$ is continuous on $[a,b]$ and differentiable on $(a,b)$, then there is a number $c$ such that $\dfrac{F(b)-F(a)}{b-a} = F'(c)$.

Why?!

Examples.

1. $\displaystyle\int_0^2 (4 - x^2)\,dx$

2. $\displaystyle\int_1^2 (x^3 - 3^x)\,dx$

3. $\displaystyle\int_{\pi/3}^{\pi/4} \sec(x)\tan(x)\,dx$

4. $\displaystyle\int_1^7 \frac{2-3x^2}{x}\,dx$

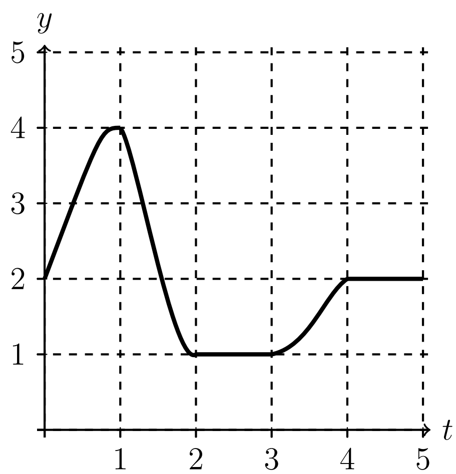
5. $\displaystyle\int_1^3 \left(\frac{3}{\sqrt{x}} - \sqrt[5]{x^2}\right)\,dx$

The Net Change Theorem

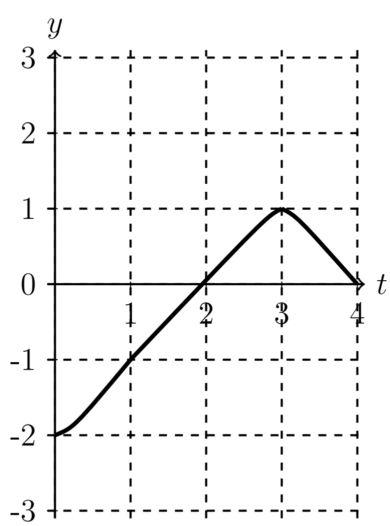
The net change in a function $f$ over an interval $[a, b]$ is given by $f(b) - f(a) = \int_a^b f'(x)\, dx$, provided that $f'$ is continuous on $[a, b]$.

6. Suppose the velocity of a particle at time $t$ is given by the function $v(t) = 6t - 12$. Determine the displacement of the particle from $t = 0$ to $t = 3$.

7. Suppose the graph below represents the *velocity* in miles per hour of a person at time $t$ hours. Estimate the displacement of the person over the time interval $[0, 5]$. (Hint: You may initially want to focus on the interval $[4, 5]$ and then think about the whole time interval.)




8. Suppose the graph below represents the *velocity* of a person in miles per hour at time $t$ hours. Estimate the *total distance traveled* over the time interval $[0, 4]$.



1. Warm-up exercise. You may use FTC Part 2.

   (a) Evaluate the integral. $\int_1^{\pi} \left(6t^2 - 2t + 3\right)\ dt$

   (b) Evaluate the integral. $\int_1^{x} \left(6t^2 - 2t + 3\right)\ dt$

   (c) Determine $\frac{d}{dx}\left(\int_1^{x} \left(6t^2 - 2t + 3\right)\ dt\right)$.

   (d) If $g(x) = \int_1^{x} \left(6t^2 - 2t + 3\right)\ dt$, determine $g(2)$ and $g'(2)$.

**The Fundamental Theorem of Calculus, Part 1**

If $f$ is continuous on $[a, b]$, then the function $F$ defined by $F(x) = \int_a^x f(t)\ dt$, $a \le x \le b$, is differentiable on $(a, b)$, and

$$F'(x) = \frac{d}{dx}\left(\int_a^x f(t)\ dt\right) = f(x).$$

2. Determine $\frac{d}{dx}\left(\int_1^{x} t^2\sqrt{1+t^2}\ dt\right)$.

3. If $g(x) = \int_x^{\pi} (\sqrt{t}\sin(t))^4\ dt$, determine $g'(x)$.

4. If $g(x) = \int_0^{x^3} e^{t^2}\, dt$, determine $g'(x)$.

5. If $g(x) = \int_{\sin(x)}^{0} t^5\, dt$, determine $g'(x)$.

6. Determine $\int_0^{3\pi/2} \sin(x)\, dx$.

7. Determine the *total area* enclosed by the $x$-axis and the curve $f(x) = \sin(x)$ over the interval $[0, 2\pi]$.

8. Determine the total area bounded by the $x$-axis, the curve $y = x^2 - x$, and the region $-2 \leq x \leq 2$.

9. Determine the total area bounded by the curve $x = y^2 - y - 2$ and the $y$-axis.

10. A particle moves along a straight line with velocity $v(t) = t^2 - 2t$.

    (a) Determine the displacement of the particle from $t = 0$ to $t = 3$.

    (b) Determine the total distance traveled by the particle from $t = 0$ to $t = 3$.