Definition. Assume that the length of the widest subinterval used in each Riemann Sum S_n goes to 0 as n goes to ∞. Then the definite integral of f on $[a, b]$, denoted by $\displaystyle\int_a^b f(x)dx$ is

$$\int_a^b f(x)dx = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x_k.$$

Notes.

⋆ If the integral $\int_a^b f(x)dx$ exists, then f **is integrable** on $[a, b]$.

⋆ We call a the lower limit of integration; b is the upper limit of integration, and together a and b are called the limits of integration.

⋆ Your WeBWorK says $||P|| \to 0$ instead of $n \to \infty$; $||P|| \to 0$ means the width of the widest subinterval (denoted by $||P||$) goes to 0.

1. The expression $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} 2c_k(1 - c_k)^2 \Delta x$ is the limit of a Riemann sum of a function f on $[0, 3]$. Write this expression as a definite integral on $[0, 3]$.

$$\int_a^b f(x)\ dx = (\text{areas of the regions above } [a, b]) - (\text{areas of the regions below } [a, b])$$

2. Use areas to evaluate the integral $\displaystyle\int_{-3}^{4} x\ dx$. Use your work to make an educated guess for the evaluation of $\displaystyle\int_a^b x\ dx$, where $a < b$.

3. Use areas to evaluate $\int_2^5 4\ dx$. Use your work to make an educated guess for the evaluation of $\int_a^b c\ dx$, where $a < b$.

4. Suppose that $\int_{-1}^{6} f(x)dx = 5$, $\int_{-1}^{6} g(x)dx = 1$, and $\int_{1}^{6} f(x)dx = 8$. Determine the following definite integrals.

(a) $\int_{-1}^{6} (g(x) - f(x))dx$

(b) $\int_{-1}^{1} f(x)dx$

Properties of the definite integral

(1) (Switch limits) $\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$, if $a > b$

(2) (Zero width) $\int_a^a f(x)\ dx = 0$

(3) (Sum and Difference Rule) $\int_a^b (f(x) \pm g(x))\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$

(4) (Constant Multiple Rule) $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$

(5) (Additivity) If c is any number in $[a, b]$, then $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$.

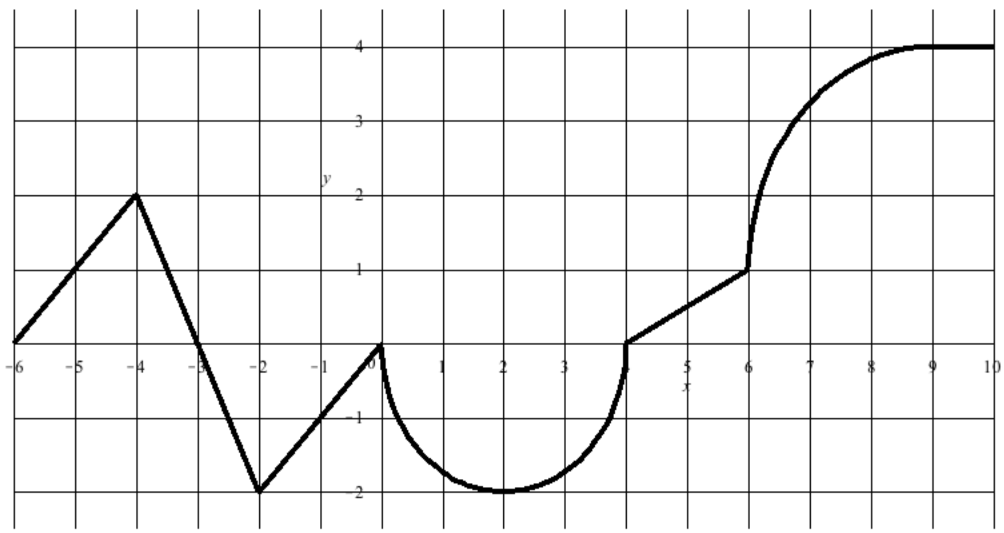
(6) (Dominance) If $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$.

(7) (Max/Min Inequality) If m and M are the min and max values of f on $[a, b]$, respectively, then $m(b-a) \leq \int_a^b f(x)\ dx \leq M(b-a)$.

> The average value of f on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

1. Determine the average value of $f(x) = \sqrt{9 - x^2}$ on the interval $[-3, 3]$.

2. The graph of f consists of straight line segments and arcs of semicircles. Determine the integrals below by interpreting each one geometrically.



(a) $\int_{-6}^{-1} f(x)\,dx$

(b) $\int_{-2}^{4} f(x)\,dx$

(c) $\int_{6}^{9} f(x)\,dx$

3. Use the graph on problem 2 above to estimate the value of the integral $\int_{-6}^{-1} f(x)\,dx$ using right endpoints and 5 rectangles of equal width.

4. For this problem, suppose that $\int_{-3}^{5} f(x)dx = -2$, $\int_{-3}^{5} g(x)dx = 1$, and $\int_{1}^{5} f(x)dx = 4$. Determine the following definite integrals.

(a) $\int_{1}^{5} f(t)dt$

(b) $\int_{1}^{1} f(x)dx$

(c) $\int_{-3}^{5} 12g(x)\ dx$

(d) $\int_{-3}^{1} f(x)dx$

(e) $\int_{1}^{-3} f(x)dx$

(f) If $h(x) \geq f(x)$, use the Dominance Property to estimate the value of $\int_{-3}^{5} h(x)\ dx$.

5. Use the Max/Min Inequality to estimate the value of $\int_{0}^{3\pi/2} \cos(x^2)\ dx$.
Hint: $-1 \leq \cos(\theta) \leq 1$ is true for all values of θ.

6. Use the Max/Min Inequality to estimate the value of $\int_{1}^{3} \frac{1}{1+x^3}\ dx$.
Hint: You need the maximum and minimum values of $f(x) = \frac{1}{1+x^3}$ on the interval $[1, 3]$.