

Old Stuff. Determine the derivative of $F(x) = \arctan(x)$.

New Perspective.

1. Determine *an* antiderivative $F(x)$ of $f(x) = x^3 + 27$.

2. Find *all* antiderivatives of $f(x) = \frac{3}{x}$.

3. Determine the *most general* antiderivative of $f(x) = \cos(x)$.

4. $\int (x^2 - 7x + \sqrt{x}) dx$

Review - Section 4.2 – Matching Derivative Property

If $f'(x) = g'(x)$ for all x in an interval (a, b) , then f and g differ by a constant on (a, b) ; that is, there exists a constant c such that $f(x) = g(x) + C$ for all x in (a, b) .

Important Information

- A function F is an antiderivative of a function f on an interval I if $F'(x) = f(x)$ for all x in I .
- The process of finding antiderivatives of a function is antidifferentiation or integration.
- The expression $\int f(x) dx$ means: find all antiderivatives of $f(x)$ with respect to x .
- The expression $\int f(x) dx$ is called the indefinite integral of f .
- In $\int f(x) dx = F(x) + C$,
 - ★ f is called the integrand
 - ★ dx tells you what your variable is. (Do not leave it off.)
 - ★ F is an antiderivative of f .
 - ★ C is the constant of integration. Every specific value of C gives you another antiderivative of f . Without the C , your answer would only be one antiderivative, not all of them. (You need the $+C$.)

5. (a) Determine $\int \frac{1}{\sqrt{x}} dx$.

(b) Determine the antiderivative $F(x)$ of $f(x) = \frac{1}{\sqrt{x}}$ with $F(0) = \pi$.

6. Complete the following table of indefinite integrals.

$\int x^n dx =$	(if $n \neq -1$)	$\int \frac{1}{x} dx =$
$\int e^x dx =$		$\int a^x dx =$
$\int \sin(x) dx =$		$\int \cos(x) dx =$
$\int \sec^2(x) dx =$		$\int \csc^2(x) dx =$
$\int \sec(x) \tan(x) dx =$		$\int \csc(x) \cot(x) dx =$
$\int \frac{1}{x^2 + 1} dx =$		$\int \frac{1}{\sqrt{1 - x^2}} dx =$

7. Determine the following indefinite integrals, using guess-and-check or using the table above.

(a) $\int \left(\frac{\pi}{\sqrt{x}} + \frac{17}{\sqrt[3]{x^4}} - \frac{1}{5\sqrt{1-x^2}} \right) dx$

(b) $\int (2^t - 3\sin(t)) dt$

Determine the following indefinite integrals.

1. $\int x \left(7 - \frac{1}{x^2} \right) dx$

2. $\int \left(e^{3x} + \frac{8}{\sqrt[5]{x^9}} \right) dx$

3. $\int \frac{3}{x^2 + 4} dx$

Solve the *differential equation* below.

4. $\frac{dy}{dx} = 7 \cos(3x) - 4 \csc(x) \cot(x)$

Solve the following *initial-value problems*.

5. $f'(x) = 1 + \cot^2(x)$, $f(\pi/4) = 7$

6. $f'(x) = \sqrt{x}(6 + 5x)$, $f(1) = 10$

7. $f''(t) = 2e^t + 3 \sin(t)$, $f'(0) = 0$, $f(\pi) = 0$

