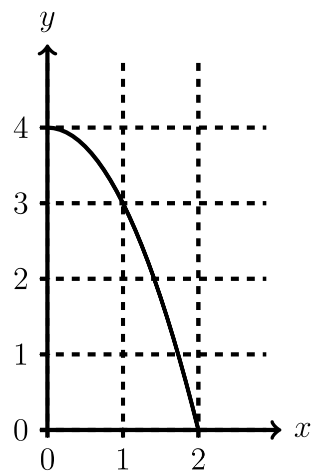
MATH 2250 - Royal Section 5.1/5.2 Handout

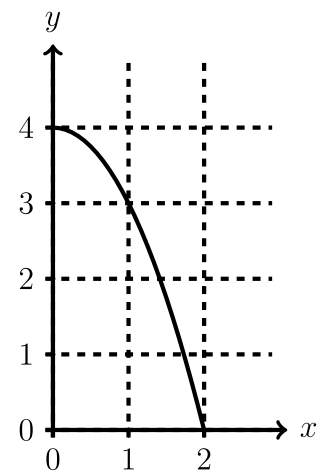
Estimating areas under curves

1. Estimate the area under the curve $f(x) = 4 - x^2$ between $x = 0$ and $x = 2$ using left endpoints and 2 rectangles of width 1, then using midpoints and 2 rectangles of width 1. Shade in the approximating rectangles on your graph.

left endpoints

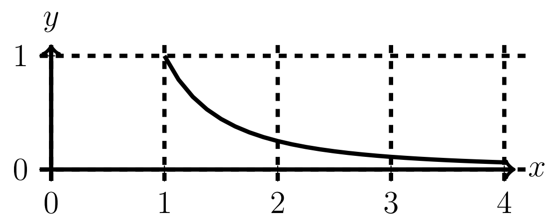


midpoints

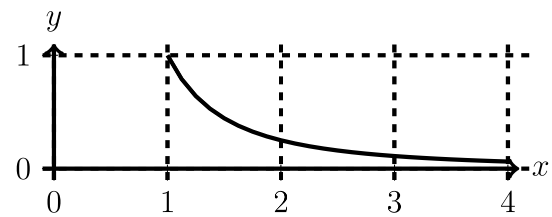


2. Estimate the area under the curve $f(x) = \dfrac{1}{x^2}$ between $x = 1$ and $x = 4$ using a *lower sum* and 5 rectangles of equal width, then by using an *upper sum* and 5 rectangles of equal width.

lower sum



upper sum



3. Estimate the area under the curve $f(x) = \sin x$ between $x = 0$ and $x = \pi$ using right endpoints and 7 rectangles of equal width.

Summation Notation

4. Write out the sum $\sum_{k=1}^{4} k^3$ in expanded form; then determine the value of the sum.

5. Write out the sum $\sum_{k=2}^{5} \frac{k}{12}$ in expanded form; then determine the value of the sum.

Definition. Let f be a continuous, **nonnegative** function defined on an interval $[a, b]$. Suppose that $[a, b]$ is divided into n subintervals of equal length $\Delta x = \frac{b-a}{n}$. Then the **exact** area of the region that lies under the graph of f on $[a, b]$ is

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x, \quad \text{where } c_k \text{ lies in the } k\text{th subinterval.}$$

6. Warm-up. Determine the approximate area under the curve $f(x) = 2x^2 + 7$ from $x = 0$ to $x = 3$ using right endpoints and 100 sub-intervals of equal length.

7. Determine the exact area under the curve $f(x) = 2x^2 + 7$ from $x = 0$ to $x = 3$, by first determining the approximate area under the curve using n sub-intervals of equal width and right endpoints.

Basic Summation Formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$