MATH 2250 - Royal Section 4.6, Applied Optimization

Note: See back of Related Rates handout for important geometric formulas. (Know them.)

Objective. Apply max/min ("optimization") procedures in an applied setting by first determining an appropriate function and domain.

Procedure for Solving Applied Optimization Problems (GPVFDO)

1. Goal. In words, what are you trying to maximize or minimize?
2. Picture. Draw a diagram and label any variables there.
3. Variables. Clearly state all of your variables and what they represent. Be sure to assign a variable to the quantity that is to be maximized or minimized. Identify any *constraints* on your variables.
4. Function. Write a function *of one variable* representing the quantity that you want to maximize or minimize. (You may need to eliminate an extra variable by using a constraint equation.)
5. Domain. Determine the domain for the function.
6. Optimization. Use calculus to determine the absolute maximum or minimum of the function (on the domain). Always verify that your critical number corresponds to an absolute max or min.

Domain. When you determine the domain for your function, use a closed interval rather than an open interval if you can. That way, you have more options in terms of optimization techniques.

Summary of Optimization Procedures

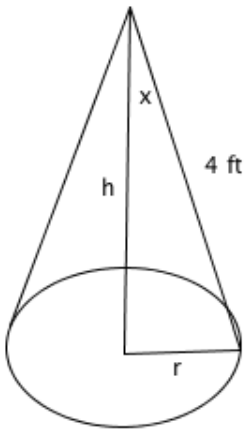
Method	Procedure	Rel or abs?

Optimization Practice Problems

1. Find two numbers whose difference is 44 and whose product is minimized. Then determine the minimum product.

2. Find the point on the curve $y = \sqrt{x+6} + 2$ which is closest to the point $(-5, 2)$.

3. Find the dimensions of the right circular cone of maximum volume having a slant height of 4 ft.



1. You are planning to make an open box from a 14 in. by 14 in. square of metal by cutting congruent squares from the four corners and then folding up the resulting flaps. What is the maximum volume of the largest such box, and what are its dimensions?

2. Find the dimensions of the rectangle of greatest possible area that can be inscribed in a semicircle of radius 6.

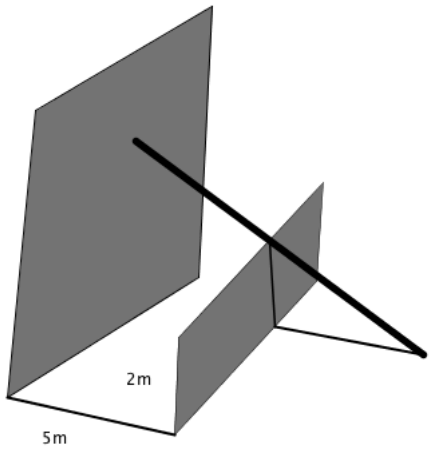
3. A fence is to be built to enclose a rectangular area of 220 square feet. The fence along three sides is to be made of material that costs \$5 per foot, and the fourth side costs \$13 per foot. Find the length L and width W (with $W \leq L$) of the enclosure that is most economical to construct.

4. A car rental agency has 24 cars (each an identical model). The owner of the agency finds that at a price of \$18/day, all the cars can be rented; however, for each \$1 increase in rental cost, one of the cars is not rented. What should the agency charge to maximize income?

1. A box with an open top has vertical sides, a square bottom, and a volume of 256 cubic meters. If the box has the least possible surface area, find its dimensions.

2. What is the smallest possible perimeter for a rectangle whose area is 36 sq. in., and what are the dimensions of the rectangle with smallest possible perimeter?

3. Find the length of the shortest beam that can be used to brace a wall if the beam is to pass over a second wall 2m high and 5m from the first wall. What is the angle of elevation of the beam?



4. A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the largest possible area of a Norman window with perimeter 35 feet?