

Warm-up: Recap of Infinite Limits and Limits at Infinity

$$1. \lim_{t \rightarrow 1} \frac{1-t}{1-\sqrt{t}}$$

Why can't we just "plug in" right away to determine this limits?

What are the seven indeterminate forms?

Section 4.5 - L'Hopital's Rule

What if we wanted to do the limits  $\lim_{x \rightarrow 0} \frac{1-e^x}{x}$  and  $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$ ?

L'Hopital's Rule

Suppose  $f$  and  $g$  are differentiable on an open interval  $I$  that contains  $a$ , with the possible exception of  $a$  itself, and that  $g'(x) \neq 0$  near  $a$ , except possibly at  $a$ . If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  is an indeterminate form of the type  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ , then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

provided the limit  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  exists or is  $\pm\infty$ .

Note: We can also apply L'Hopital's Rule to one-sided limits and to  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$  and  $\lim_{x \rightarrow -\infty} \frac{f(x)}{g(x)}$

whenever we have an indeterminate form of type  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ .

Examples. Whenever you apply L'Hopital's rule, label the work with LH and the type of indeterminate form (e.g.  $\frac{0}{0}$ ). However, don't write that your limit is "equal" to the indeterminate form.

$$1. \lim_{x \rightarrow 0} \frac{1-e^x}{x}$$

$$2. \lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$$

$$3. \lim_{x \rightarrow \pi/2} \frac{x(1 - \sin(x))}{\cos(x)}$$

$$4. \lim_{x \rightarrow 0} \frac{2^x - 11^x}{x}$$

$$5. \lim_{t \rightarrow \infty} 4t^3 e^{-5t}$$

$$6. \lim_{x \rightarrow \pi/2} \frac{\cos(3x)}{\cos(5x)}$$

$$7. \lim_{x \rightarrow \infty} \frac{e^{2x}}{5x^2}$$

$$8. \lim_{x \rightarrow 0^+} (\csc(x))^{\sin(x)}$$

$$1. \lim_{x \rightarrow 0} \frac{5x^2}{1 - \cos(x)}$$

$$2. \lim_{x \rightarrow \infty} \frac{e^x - x^2 + 2}{x^2}$$

$$3. \lim_{x \rightarrow -\infty} \frac{e^{2x}}{5x^2}$$

$$4. \lim_{x \rightarrow 1^+} \frac{\tan(x-1)}{\sqrt[3]{x-1}}$$

$$5. \lim_{x \rightarrow 1^+} \frac{\tan(x-1)}{(x-1)^2}$$

$$6. \lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$$

$$7. \lim_{x \rightarrow 0^-} (\csc(x) - \cot(x))$$

$$8. \lim_{x \rightarrow \infty} \left( \frac{12x}{12x + 7} \right)^{3x}$$

$$9. \lim_{x \rightarrow \infty} (1 + 3/x)^{5x}$$

$$10. \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} (\tan(x))^{\cos(x)}$$

$$11. \lim_{x \rightarrow 0^+} (\sin(x))^{\tan(x)}$$