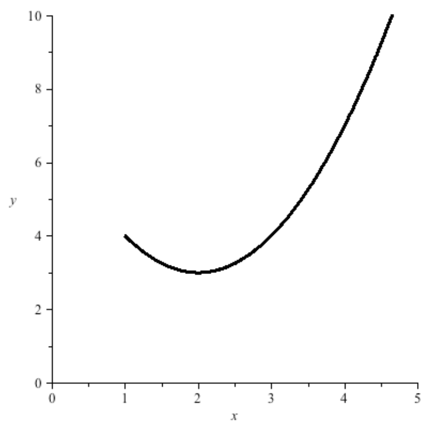
Definition. Suppose f is differentiable on an open interval I.

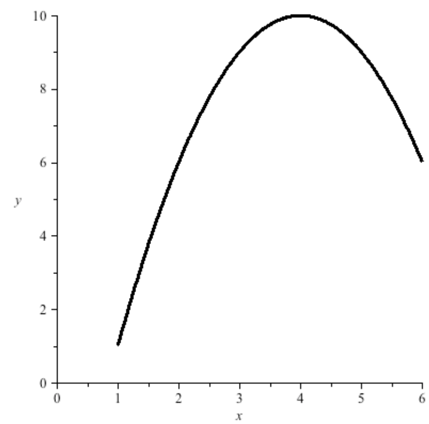
- The graph of f is concave upward on I if f' is increasing on I.
- The graph of f is concave downward on I if f' is decreasing on I.

Concave Up



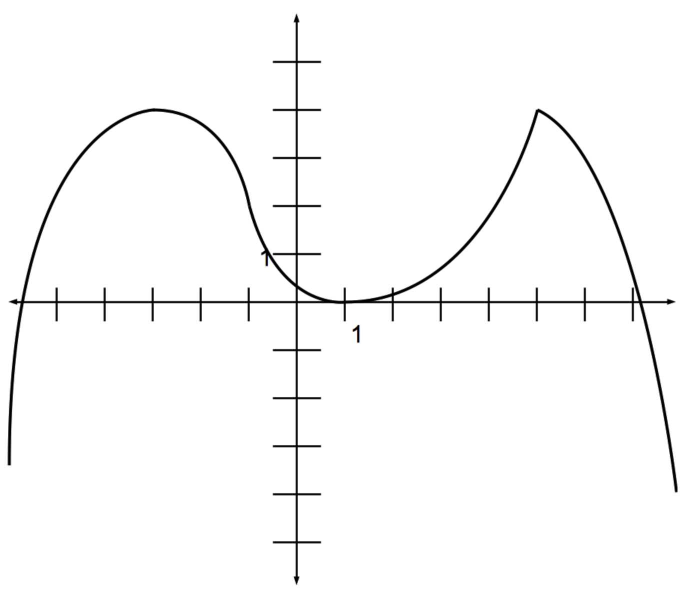
up like a *cup*

Concave Down



down like a *frown*

1. Determine intervals where the graph of $y = f(x)$ below is concave up and concave down.



Definition. Suppose f has a tangent line at the point $(c, f(c))$ and the concavity of f changes there (from up to down, or from down to up). Then $(c, f(c))$ is called an inflection point.

2. Determine all inflection points of the graph of f above.

Concavity Test Suppose f has a second derivative on an open interval I.

1. If $f''(x) > 0$ on I, then the graph of f is concave upward on I.
2. If $f''(x) < 0$ on I, then the graph of f is concave downward on I.

Important Idea: The second derivative can only change sign where __________________________.

3. Suppose f is a function whose second derivative $f''(x) = x^2(x+5)(x-4)$ is defined on $(-\infty, \infty)$. Determine intervals on which f is concave up and concave down, and find all inflection points.

4. Determine intervals where $f(x) = e^{-x^2/2}$ is concave up and concave down, and find all inflection points.

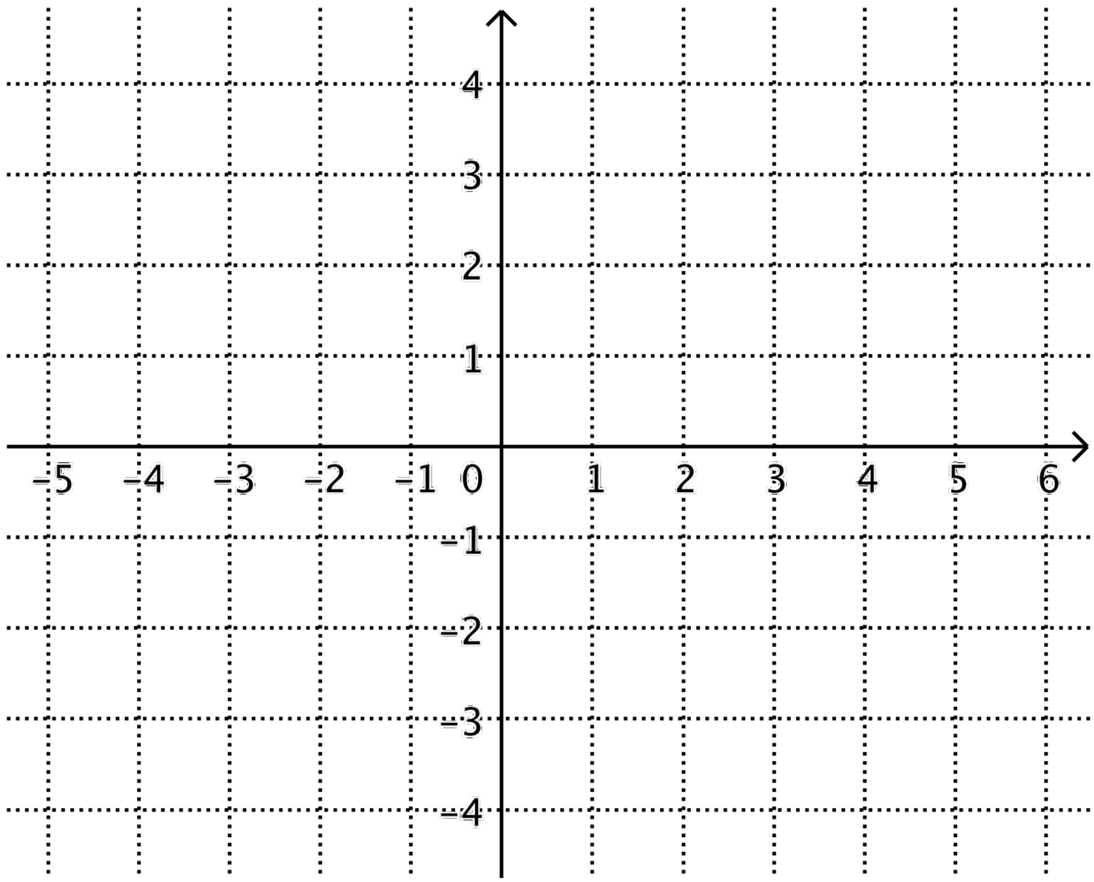
5. Determine intervals where $f(x) = \dfrac{x}{x^2 - 9}$ is concave up and concave down, and find all inflection points.

1. Sketch the graph of a function which satisfies the following:

   Domain of f: $(-\infty, -1) \cup (-1, \infty)$

interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 1)$	$(1, 3)$	$(3, \infty)$
beh of f	decr	incr	incr	decr	incr

interval	$(-\infty, -1)$	$(-1, 2)$	$(2, 3)$	$(3, \infty)$
concavity	CU	CD	CU	CD



<u>The Second Derivative Test</u> Suppose that f has a continuous second derivative on an interval (a, b) containing a critical number c of f. (This means that $f'(c) = 0$.)
(a) If $f''(c) > 0$, then f has a relative minimum at c.
(b) If $f''(c) < 0$, then f has a relative maximum at c.
(c) If $f''(c) = 0$, then the test is inconclusive.

2. Use the Second Derivative Test to determine the relative maxima and minima of $f(x) = 2x^3 + 3x^2 - 12x + 6$.

3. Determine the relative maxima and minima of $f(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 7$.

4. For this problem, use $f(x) = 2xe^x - x^2e^x$.

(a) Use the *first* derivative test to find the relative extrema of f.

(b) Use the *second* derivative test to find the relative extrema of f.

(c) Which technique is simpler for this example? Why?

5. Let f be a function such that $f''(x)$ is defined for all real numbers. A table of some values of f' is given below. Assume that f' is continuous and either always increasing or always decreasing between consecutive values of x shown in the table. Answer the following questions; if there is not enough information to answer the question, put "not enough info."

x	2	3	4	6	9	11
$f'(x)$	4	1	0	2	0	-4

(a) Find the x-coordinates of all critical points of $f(x)$ on the interval $2 < x < 11$.

(b) Find the x-coordinates of all local minima of $f(x)$ on the interval $2 < x < 11$.

(c) Find the x-coordinates of all inflection points of $f(x)$ on the interval $2 < x < 11$.

(d) Find all values of x at which $f(x)$ attains its global maximum on the interval $2 \le x \le 11$.