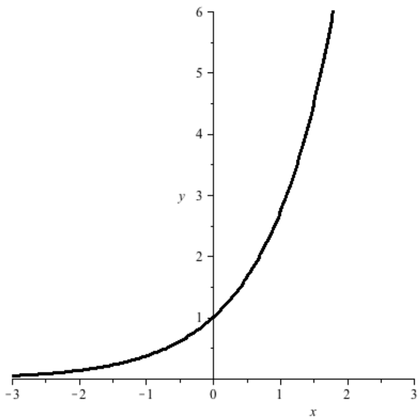
MATH 2250 Section 4.3 - Increasing/Decreasing Functions and First Derivative Test

Definitions

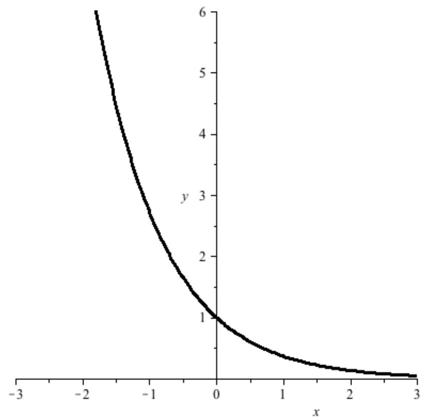
A function f is an increasing function if the y-values on the graph increase as you go from left to right.

Example. $f(x) = e^x$



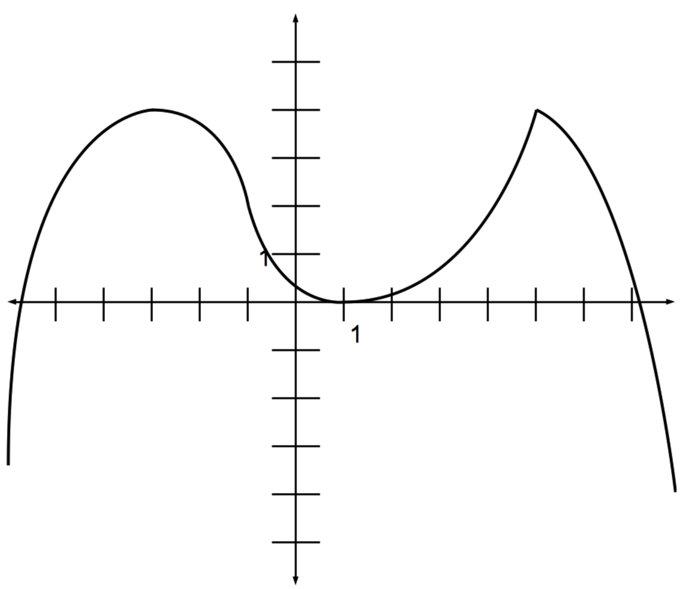
A function f is an decreasing function if the y-values on the graph decrease as you go from left to right.

Example. $f(x) = e^{-x}$



Most functions switch back and forth from increasing to decreasing.

Example. The graph of f is below. Assume that it includes all of the relevant information about f, and that the domain of f is $(-\infty, \infty)$. Determine intervals on which f is increasing and decreasing.



Increasing/Decreasing Test Suppose f is differentiable on an open interval (a, b).

- If $f'(x) > 0$ for all x in (a, b), then f is increasing on (a, b)
- If $f'(x) < 0$ for all x in (a, b), then f is decreasing on (a, b).
- If $f'(x) = 0$ for all x in (a, b), then f is constant on (a, b).

Key observation: If f' changes sign at c, then either $f'(c) = 0$ or $f'(c)$ dne, so c is a critical number of f.

1. Let $f(x)$ be a continuous function whose derivative is $f'(x) = x(x+3)^2(x-5)$. Determine intervals where $f(x)$ is increasing and decreasing.

2. Find intervals where $f(x) = 2x^3 + 3x^2 - 12x + 6$ is increasing and decreasing.

The First Derivative Test Suppose c is a critical number of a continuous function f in the interval (a, b) and suppose that f is differentiable at every number in (a, b) with the possible exception of c itself.

- If $f'(x) > 0$ on (a, c) and $f'(x) < 0$ on (c, b), then f has a relative max at c.
- If $f'(x) < 0$ on (a, c) and $f'(x) > 0$ on (c, b), then f has a relative min at c.
- If $f'(x)$ has the same sign on (a, c) and (c, b) then f does not have a relative extremum at c.

3. Determine relative and absolute extrema of the function whose graph is on the previous page.

4. Find intervals where $g(u) = u^{-1}e^u$ is increasing and decreasing, and identify relative extema.

1. Find intervals where $f(x) = \dfrac{\ln(x)}{x^3}$ is increasing and decreasing.

2. Determine relative and absolute extrema of $f(x) = x^{1/5}(x - 10)$.

3. Consider functions f satisfying all of the following conditions:

- $f(x)$ is differentiable on the interval $0 < x < 8$
- The critical points of $f(x)$ in the interval $0 < x < 8$ are $x = 2, 4,$ and 6. ($f(x)$ has no other critical points in this interval.)
- The table below shows some values of $f(x)$ and of its derivative $f'(x)$.

x	1	3	5	7
$f(x)$	3	6	11	0
$f'(x)$	-1	?	?	-1

For each of the statements below, decide whether the statement is true for all functions f satisfying all of the conditions described above, for SOME of these functions f, or for NONE of these functions f.

(a) $f(x)$ has a local minimum at $x = 2$

(b) $f'(3) > 0$

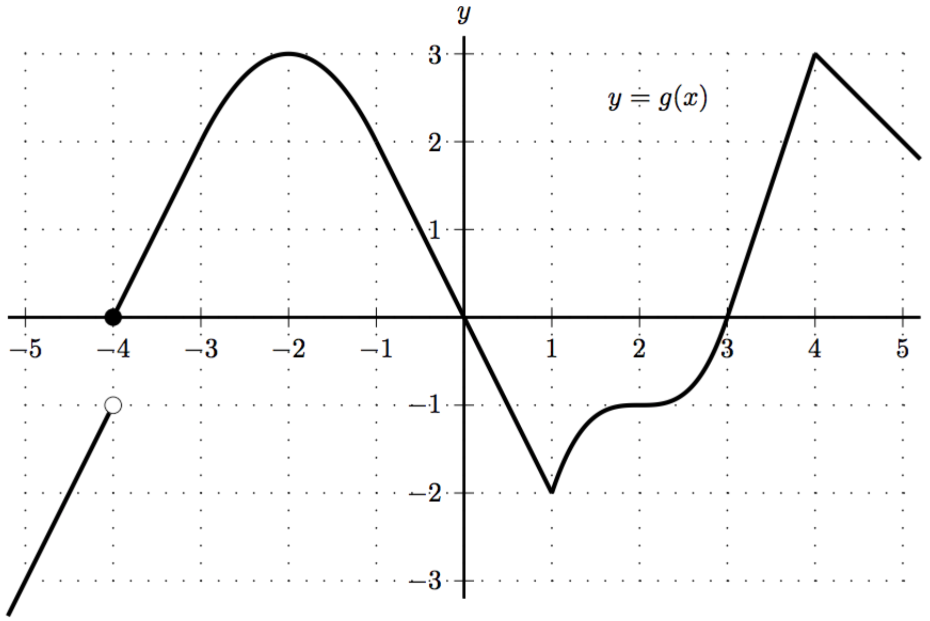
(c) $f(x)$ has a local maximum at $x = 4$

(d) There is exactly one value of a with $3 < a < 7$ such that $f(x)$ has a local maximum at $x = a$.

4. Find the relative extrema of $f(x) = x^4 - 4x^3$.

5. Find the relative extrema of $f(x) = x^{2/3}(x - 5)$.

6. A portion of the graph of $y = g(x)$ is shown below.



On the axes below, sketch the graph of $y = g'(x)$. Be sure you pay close attention to each of the following:

- where g' is defined
- the value of $g'(x)$ near each of $x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$
- the sign of g'
- where g' is increasing/decreasing/constant