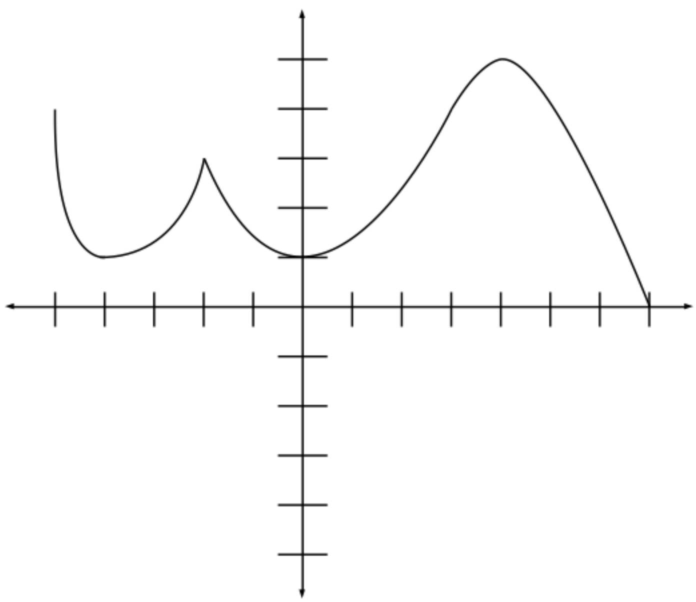
Definitions - Absolute Maxima and Minima

- A function f has an *absolute maximum* (or global maximum) at c if $f(c) \geq f(x)$ for all x the domain of f. The number $f(c)$ is called the maximum value.
- Similarly, f has an *absolute minimum* (or global minimum) at c if $f(c) \leq f(x)$ for all x in the domain of f. The number $f(c)$ is called the minimum value.
- The max. and min. values of f are called the *extreme values (extrema)* of f.

1. What are the extreme values of the function f whose graph is below? (Assume each tick mark represents one unit and the domain is $[-5, 7]$.)



2. Let

$$f(x) = \begin{cases} -\sqrt{9-x^2} & \text{if } -3 \leq x \leq 0 \\ \sqrt{9-x^2} & \text{if } 0 < x \leq 3. \end{cases}$$

Sketch the graph of the function and find its absolute maximum and minimum values.

Definitions - Relative (Local) Maxima and Minima

- A function f has a *relative maximum* (or local maximum) at c if $f(c) \geq f(x)$ for all values of x near c and in the domain of f. The number $f(c)$ is called a relative maximum value.
- Similarly, f has a *relative minimum* (or local minimum) at c if $f(c) \leq f(x)$ for all values of x near c and in the domain of f. The number $f(c)$ is called a relative minimum value.

3. (a) Determine all *relative* maxima and minima of the function on #1.

(b) Look at the tangent lines at the corresponding points. What do you notice?

Key idea. If we look for values c where $f'(c) = 0$ or $f'(c)$ dne, we will get some numbers c where f ________________________ have a relative maximum or minimum.

Definition. A *critical number (or critical point)* of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

4. Determine the critical numbers of the function on #1.

5. Determine the critical numbers of the function $f(x) = x \ln(x)$.

The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Finding Maxima and Minima on Closed Intervals

If f is continuous on the closed interval $[a, b]$, to find the absolute maximum and minimum values of f:

(1) Find the critical numbers of f that lie in (a, b).

(2) Find the values of f at all of the critical numbers of f in (a, b), and find the values of f at the endpoints of the interval, i.e. at a and at b.

(3) The largest of the values of f in Step 2 is the absolute maximum value. The smallest of the values of f in Step 2 is the absolute minimum value.

6. Determine the absolute maximum and absolute minimum values of $f(x) = 3x^2 - 18x + 7$ on the interval $[1, 4]$.

1. Determine the absolute maximum and absolute minimum values of $f(x) = x + \dfrac{4}{x}$ on the interval $[1, 5]$.

2. Determine the absolute maximum and absolute minimum values of $f(x) = \dfrac{x}{x^2 + 4}$ on the interval $[0, 3]$.

3. Consider the function $f(x) = x^{3/2} - 4\sqrt{x}$

   (a) Find the absolute minimum and absolute maximum of $f(x)$ on the interval $0 \leq x \leq 5$.

   (b) Find the absolute extrema of f.

4. Determine all absolute extrema of the function $g(t) = e^{t^2} + e^{-t^2}$.

5. Consider the function $f(x) = \frac{1}{20}x^5 - \frac{1}{12}x^4 - x^3 + 7$. Determine the absolute minimum and absolute maximum of the derivative of f on the interval $[-3, 1]$.

6. Determine all absolute extrema of the function $f(x) = |x| - x^2 - 2x$.

7. Determine the absolute extrema of the function $f(x) = \sin(x) + \cos(x)$.