MATH 2250 - Royal Section 3.3: Differentiation Rules

From now on, use the shortcuts unless you see "use the definition of the derivative" in the directions. If you do see "use the definition of the derivative," you must use $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to get credit for your answer.

Examples.

1. Use *differentiation rules* to differentiate the following functions. Write the names of the rules you use.
   (a) $f(x) = 27$

   (b) $f(x) = x^5$

   (c) $y = \sqrt[3]{x}$

   (d) $A = \pi r^2$

   (e) $y = x^2 - x + \dfrac{1}{x^3}$

   (f) $f(x) = \pi e^x + e^\pi$

2. Determine $\dfrac{d}{dx}\left(\dfrac{2x^7 - \sqrt{x} + 12}{x^2}\right)$.

3. Determine $f''(x)$ if $f(x) = \sqrt[5]{x^4} - \dfrac{1}{\sqrt{x}}$.

4. Determine $f^{(4)}(x)$ if $f(x) = -3x^5 - 7x^4$.

5. For this problem, use the curve $y = \frac{1}{3}x^3 - 2x + 5$.

   (a) Find the points on the graph of the curve at which the tangent line is parallel to $y = 2x + 3$.

   (b) What is the smallest slope for this curve? At what point(s) does the curve have this slope?

6. Determine an equation for the line perpendicular to the tangent line to the curve $y = -2 + x^2$ at $(1, -1)$.

Why do these rules work? We'll *prove* a couple of them are always true.

7. Use the definition of the derivative to prove the Sum Rule for differentiation.

8. Use the definition of the derivative to prove the Constant Multiple Rule for differentiation.

Warm-up/Review

- Determine $f'(x)$ for $f(x) = x(-x + 17)$.

- Determine $\dfrac{du}{dy}$ for $u = \dfrac{y+3}{\sqrt[3]{y}}$.

Examples.

1. Differentiate the following functions.
   (a) $y = (x^3 + x^2 + 5)(3 - x + \sqrt{x})$

   (b) $f(x) = x(4 + 3e^x)$

   (c) $f(x) = \dfrac{e^x + 1}{x^2 - 5}$

   (d) $y = \dfrac{x^2 + 3}{x - 1} + \dfrac{1}{x^2}$

   (e) $f(x) = \dfrac{x^2 - xe^x + 17}{7 - x^5 + 12x^2}$

2. $f(t) = \dfrac{-2t}{3t+7}$.

   (a) Determine $f'(t)$.

   (b) Determine an equation of the tangent line at $(-1, f(-1))$.

   (c) Determine $f''(5)$.

3. If $y = x^3 e^{x+2}$, determine $\left.\dfrac{dy}{dx}\right|_{x=-1}$.

4. Use the table below to answer the following.

$f(3) = 4$	$f'(3) = -2$
$h(3) = -1$	$h'(3) = \pi$

   (a) If $y = f(x) \cdot h(x)$, determine $\left.\dfrac{dy}{dx}\right|_{x=3}$.

   (b) If $g(x) = \dfrac{f(x)}{h(x)}$, determine $g'(3)$.

5. Use the definition of the derivative to prove the product rule for differentiation.